Interference phenomena in waveguides with two corrugated boundaries

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Abstract. Waveguide structures with two corrugated interfaces are considered. It is shown theoretically and experimentally that these structures can form the basis for a highly-efficient unidirectional grating coupler for slab waveguides. The anomalous light reflection from waveguides with two corrugated boundaries is also considered. The possibility of using corrugated waveguides for frequency stabilization of a semiconductor laser is demonstrated.

1. Introduction

The formation of diffraction gratings on optical waveguides leads to suitable coupling devices for integrated optics. Planar geometry makes the devices compact and convenient in practical applications. The theoretical description of the corrugated waveguides and the problems in their practical use is well developed at present [1–3]. As the object of theoretical investigation, a corrugated ion-exchanged waveguide or a thin-film waveguide with one corrugated boundary has usually been chosen; real devices, however, frequently have two corrugated interfaces. For technological convenience, thin-film waveguides are often deposited on a previously corrugated substrate. To enhance interaction between the waveguide mode and the grating, the surfaces of ion-exchanged waveguides are sometimes coated with a layer of high refractive index. In general, the structure is corrugated on both boundaries with equal period, but with different phase and amplitude. Since the wave-vectors of these diffraction gratings are equal, the waveguide mode diffracts in coinciding directions on each grating (in the grating coupler each corrugation contributes to radiation towards both the substrate and superstrate). To estimate the result of the waveguide mode diffraction, the contribution of each grating into diffracted radiation must be superposed, taking into account the phase shift between the diffracted waves. The result of the total influence of both gratings upon the diffraction phenomena in corrugated waveguides therefore has interferometric properties, and is defined mainly by the relative grating position. The present paper deals with the use of such interference in the operation of various devices based on corrugated waveguides.

We shall consider here structures such as combined waveguides (thin film on the graded-index waveguide) and multilayer slab waveguides. Diffraction problems in these structures should be solved taking into account the mode field distribution to obtain the mode field at the corrugated interfaces. We paid attention to deformation of the mode profile, when varying any of the structure parameters.

The problems discussed in this paper are as follows. We shall first consider light radiation from the corrugated waveguide structures described above, and then anomalous light reflection from the surface of these structures. The former problem yields an analytical solution and hence a rather general analysis, whereas the latter
one leads to an extended set of equations which can only be solved numerically. On studying the process of light radiation from a corrugated waveguide in detail, we predict some general specific features of anomalous reflection that we confirm by particular numerical calculations.

2. Radiative losses in the combined waveguides

We defined a combined waveguide as a graded-index waveguide whose surface is coated with a thin homogeneous film (figure 1 (a)). A typical profile of dielectric permeability \( n^2(z) \) is represented in figure 1 (b) \((n_1, n_2\text{ and } n_3)\) are the refractive indices of superstrate, film and substrate respectively, \( h \) is the film thickness and \( n_3 \) is the refractive index of the graded-index waveguide near the interface 'waveguide-film'. No specific conditions are imposed on the \( n^2(z) \) profile, except that the change in refractive index be gradual. We also presume that there is no reflection of radiation waveguide modes from such a gradient of the refractive index, and that in the small region \(-h-\sigma<z<-h\), where \( \sigma \) is the typical grating depth, the refractive index differs only slightly from \( n_3 \), so that the radiated wave here can be regarded as the plane one.

A transverse electric (TE) mode field distribution is assumed to be known in the unperturbed combined waveguide (in the absence of corrugation)

\[
E_x = f(z) \exp(ikn^*x),
\]

where \( k = 2\pi/\lambda, \lambda \) is the light wavelength in vacuum, \( n^* \) is the effective refractive index of the waveguide mode), and the normalization is such that

\[
\int_{-\infty}^{+\infty} |f(z)|^2 \, dz = 1.
\]

In the case of corrugated boundaries, the 'film–air' and 'film–waveguide' interfaces

![Figure 1](image-url)

Figure 1. (a) Corrugated combined waveguide. (b) The \( n^2(z) \) profile \((n \text{ is the refractive index})\) of the combined waveguide. (c) Field distribution of the combined waveguide mode.
are defined by
\[ z_1(x) = \sigma_1 \sin Kx, \] (3)
\[ z_2(x) = -h + \sigma_2 \sin (Kx + \varphi) \] (4)
where \( \sigma_1 \) and \( \sigma_2 \) are the grating depths, \( \varphi \) is their relative phase shift, \( K = 2\pi/\Lambda \), \( \Lambda \) is the corrugation period. The total field is a superposition of the waveguide mode field (1) and the fields \( E_r \) of radiative modes, which in the first-order approximation on \( \sigma_1, \sigma_2 \) can be considered to be homogeneous plane waves in the film, superstrate, substrate and graded-index waveguide near the film boundary
\[
E_r = \begin{cases} 
A \exp \left\{ ik \left[ g_1 z + \left( n^s - \frac{1}{\Lambda} \right) x \right] \right\}, & z > z_1(x), \\
B \exp (ikg_2 z) + C \exp (-ikg_2 z) \exp \left[ ik \left( n^s - \frac{1}{\Lambda} \right) x \right], & z_1(x) > z > z_2(x), \\
D \exp \left\{ ik \left[ -g_3 z + \left( n^s - \frac{1}{\Lambda} \right) x \right] \right\} \exp (-ikg_3 h), & -h - \sigma_2 < z < z_2(x),
\end{cases}
\] (5)
\[
g_j = \left[ n_j^2 - \left( n^s - \frac{1}{\Lambda} \right)^2 \right]^{1/2}, \quad j = 1, 2, 3 \ldots
\] (6)
The grating and waveguide parameters are assumed to provide radiation into air and substrate, that is \( g_j \) in equation (6) is real. The total field \( E = E_r + E_z \) must be continuously differentiable (in the case of TE-mode). This requirement is automatically satisfied except at the film boundaries, where the field \( E \) is continuously differentiable if
\[
A = B + C,
\]
\[
g_1 A - n_1^2 \frac{k \sigma_1}{2} E_1 = g_2 (B - C) - n_2^2 \frac{k \sigma_2}{2} E_1,
\]
\[
D = B \exp (-ikg_2 h) + C \exp (ikg_2 h),
\]
\[
-g_3 D - n_3^2 \frac{k \sigma_2}{2} \exp (-i\varphi) E_2
\]
\[= g_2 [B \exp (-ikg_2 h) - C \exp (ikg_2 h)] - n_2^2 \frac{k \sigma_2}{2} \exp (-i\varphi) E_2,
\] (7)
where \( E_1 = f(0), E_2 = f(-h) \) is the mode field at the film boundaries. Having solved the set of equations (7) with respect to \( A, B, C \) and \( D \) one can find the radiative losses, associated with radiation into air (the medium of refractive index \( n_1 \)) \( \alpha_1 = (g_1/n^s)|A|^2 \) and substrate \( \alpha_2 = (g_3/n^s)|D|^2 \), given by
\[
\alpha_1 = \frac{g_1}{n^s G} \left\{ \left( \frac{\beta_3}{g_2} S_1 - S_2 \right)^2 + 2 \frac{\beta_3}{g_2} S_1 S_2 \left[ 1 - \cos (\varphi - \psi_3) \right] \right\},
\] (8)
\[
\alpha_2 = \frac{g_3}{n^s G} \left\{ \left( S_1 - \frac{\gamma_1}{g_2} S_2 \right)^2 + 2 \frac{\gamma_1}{g_2} S_1 S_2 \left[ 1 - \cos (\varphi + \psi_1) \right] \right\}.
\] The total radiative losses are defined by the following formula
\[
\alpha = \alpha_1 + \alpha_2.
\] (9)
The new terms in relation (8) are defined as follows:
\[ G = g_2^2 \left[ (g_3^2 + g_1 g_3)^2 - (g_2^2 - g_3^2)(g_2^2 - g_3^2) \cos^2 \gamma \right], \quad \gamma = \frac{1}{2} \frac{g_1}{g_2} \kappa \sigma_1 (n_2^3 - n_1^3), \]
\[ S_1 = \frac{1}{2} E_1 \kappa \sigma_1 (n_2^3 - n_1^3) \] normalized grating depths
\[ S_2 = \frac{1}{2} E_2 \kappa \sigma_2 (n_2^3 - n_3^3) \]
\[ \gamma_j = (g_j^2 \cos^2 \gamma + g_j^2 \sin^2 \gamma)^{1/2}, \quad j = 1, 3, \]
\[ \psi_j = \left[ \arccos \left( \frac{g_j^2 \cos \gamma}{\gamma_j} \right) \right] \text{sign}(\sin \gamma), \quad j = 1, 3. \]

(10)

Formulae (8) and (10) provide comprehensive information on the radiative losses in a corrugated combined dielectric waveguide.

In the case of a one-side corrugated waveguide \((\sigma_2 = 0)\) and for a large asymmetry of the waveguide
\[ (n_2^3 - n_3^3) \ll (n_2^3 - n_1^3) \]
the relationship between the radiation into air and substrate coincides with the well known result [4]
\[ \frac{\alpha_1}{\alpha_2} = 1 \frac{g_2}{g_3}, \]
that is, for glass-based waveguides not more than 40% of radiation escapes into the air. To enhance this portion, blazed dielectric gratings are generally proposed [3]. However, the technology for constructing such a grating is rather complicated, and is applicable only to a small group of materials. Moreover, tough requirements for the accuracy of construction are the main obstacle to wide use of efficient blazed grating couplers.

If couplers with two corrugated boundaries are obtained, then (as is seen from formula (8)), a radiative loss distribution between air and substrate is reached with the symmetrical smooth (sinusoidal) profile gratings.

Note that for \(\alpha_2 = 0\) (for mode coupling only into air) two conditions should be satisfied
\[ \frac{\sigma_1}{\sigma_2} = \frac{E_1(n_2^3 - n_1^3) \gamma_1}{E_1(n_2^3 - n_1^3) \gamma_2}, \]
\[ \varphi = \psi_1. \]

(13) (14)

This means that the grating depths should be matched such that the contribution of each grating into radiation towards the substrate is equal, and the phase \(\varphi\) is chosen such that the wave radiated into substrate from each grating is in the opposite phase. Conditions (13) and (14) are not too critical, as at the minimum of \(\sigma_2(\sigma_1, \sigma_2, \varphi)\) we have
\[ \frac{\partial \sigma_2}{\partial \sigma_1} = \frac{\partial \sigma_2}{\partial \sigma_2} = \frac{\partial \sigma_2}{\partial \varphi} = 0. \]

One of the most probable ways for fabricating the aforementioned device is waveguide deposition by tilted molecular beam or by sputtering, followed by the etching with a tilted ion beam. The grating depth remains effectively constant if the thickness of the sputtered film is in the range of several hundreds of Ångström.
units, and for numerical modelling it is therefore reasonable to choose \( \sigma_1 = \sigma_2 \). As the film thickness is increased, the mode distribution profile \( f(z) \) is changed. This aspect of the problem should be specially mentioned, namely that deposition of the film of high refractive index increases the waveguide mode field value at the boundary and consequently the radiative losses.

As a numerical example we have chosen the waveguide of parabolic profile with thin film on top.

\[
n^2 = \begin{cases} 
  n_1^2, & z > 0, \\
  n_2^2, & 0 > z > -h, \\
  n_3^2 - (n_3^2 - n_4^2) \left[ b \left( \frac{z+h}{d} \right)^2 - \frac{z+h}{d} \right], & -h > z > -h - d_0, \\
  n_4^2, & z < -h - d_0,
\end{cases}
\]

where \( n_1 = 1 \), \( n_2 = 2 \), \( n_3 = 1.542 \), \( n_4 = 1.512 \), \( kd = 20 \), \( \lambda = 0.6328 \, \mu\text{m} \), \( \Lambda = 0.4 \, \mu\text{m} \), \( \sigma_1 = \sigma_2 = 0.01 \, \mu\text{m} \) and \( b = 0.64 \).

Mode profiles of this waveguide for different film thickness are illustrated in figure 2. As the film thickness increases, the graded-index waveguide mode (curve 1) continuously deforms and transforms to the film waveguide mode (curve 4). The field at the surface grows significantly (figure 3(a)) resulting in an increase in radiative losses (figure 3(b)).

The aforementioned interference phenomena lead to a considerable radiative mode energy redistribution between the air and substrate. If phase \( \varphi \) is chosen for each film thickness according to (14), then for film thickness 400–800 Å the portion of light irradiated into air increases to 80%, whereas when \( \varphi = 0 \) it is 40% (figure 3(b)). It should be noted that the relationship of the grating depths is not optimal from the point of view of minimization of radiation into the substrate. If \( \sigma_1/\sigma_2 \) is chosen according to (13), for example, \( \sigma_1 = 60 \, \AA \), \( \sigma_2 = 165 \, \AA \), \( \varphi = -1.63 \) rad, then at the same film thickness \( kh = 0.8 \) we have \( \alpha = \pi = 110 \, \text{cm}^{-1} \), that is, unidirectional radiation coupling into air takes place.

![Figure 2](image-url)  

Figure 2. The combined waveguide mode profiles. The waveguide parameters are indicated in the text. Film thickness \( kh = 0 \) (curve 1), 0.3 (curve 2), 0.6 (curve 3), 0.9 (curve 4), with the second mode profile at \( kh = 0.9 \) (curve 4'). Dashed lines indicate film–air interface positions.
3. Particular cases: graded-index and step-index waveguides

Formulae (8) and (9) assume asymptotic transfer to the case of the graded-index waveguide. Indeed, assuming that $\sigma_1 = \sigma_2 = \sigma$, $\varphi = 0$, $h = 0$ and $E_1 = E_2 = E_0$ the radiative losses are

$$\alpha_{\text{diff}} = \frac{E_0^2[(n_3^2 - n_1^2)k\sigma/2]^2}{n^w(g_1 + g_3)} \quad (15)$$

and radiation distribution between air and substrate is defined by formula (12).
Note that formula (15) is not suitable for practical use owing to the necessity for calculation of the waveguide field. The more suitable case, which is expressed via measurable parameters, is presented in [5].

It is more important to consider another particular case, that of a thin-film waveguide. In this case all assumptions on the interference phenomena in a combined waveguide remain valid. Formulae for radiative losses can be written in the more convenient form [6], since the mode field on the boundaries of the waveguide layer can be expressed via other waveguide parameters:

\[
E_1 = \left( \frac{2(n_2^2 - n_{\ast}^2)}{h^n(n_2^2 - n_1^2)} \right)^{1/2},
\]

\[
E_2 = \left( \frac{2(n_2^2 - n_{\ast}^2)}{h^n(n_2^2 - n_3^2)} \right)^{1/2} (-1)^m,
\]

where \( m = 0, 1, 2 \ldots \) is the waveguide mode number and

\[
h^n = h + \frac{\lambda}{2\pi} \left[ (n_{\ast}^2 - n_1^2)^{-1/2} + (n_{\ast}^2 - n_3^2)^{-1/2} \right]
\]

is the effective thickness of the waveguide. Since it is fairly simply realized, this case is of some practical interest.

4. Experimental realization of the undirectional coupling out of a dielectric waveguide

To demonstrate air-to-substrate energy redistribution we have carried out the following experiment. An Nb_2O_5 (\( n_2 = 2.3 \)) film waveguide has been sputtered over a corrugated (\( \sigma_2 = 700 \) Å \( \lambda = 0.34 \) μm) glass (\( n_3 = 1.512 \)) substrate such that the interface ‘waveguide–air’ repeated the surface of the substrate (\( \sigma_1 = \sigma_2, \varphi = 0 \)). The waveguide was then exposed to ion-beam etching with an oblique incidence beam. After a time interval the process was halted, the sample extracted from vacuum chamber and the \( \alpha_1/(\alpha_1 + \alpha_2) \) ratio measured.

The previously developed mathematical model of profile erosion under ion-beam etching for oblique incidence beam [7] predicts, along with reduction of the film thickness, a shift of corrugation in the direction of gratings’ wave-vector with respect to its initial position, namely the phase shift between the upper and lower corrugations. Using this model one can numerically predict the phaseshift dependence on the etching time, and hence calculate the evolution of the ratio \( \alpha_1/(\alpha_1 + \alpha_2) \) in the course of etching from equations (8)–(10), taking the diminishing of \( h \) into account. The calculated dependence and measured ratios of \( \alpha_1/(\alpha_1 + \alpha_2) \) against etching time are presented in figure 4 (b), showing considerable coincidence. This confirms that the formulae (8)–(10) for air-to-substrate energy redistribution, and the model predicting the phase shift, are valid. Applying them to another sample, we calculated the etching time for achieving maximum redistribution into air. Having halted the etching process at the calculated time we obtained a unidirectional grating coupler with \( \alpha_1/(\alpha_1 + \alpha_2) \approx 0.97 \). The waveguide thickness was \( h = 0.32 \) μm, phase shift between the corrugations \( \varphi = 0.96 \) rad, the ratio between the grating depths \( \sigma_1/\sigma_2 \approx 0.6 \). The waveguide supported two modes; the results above have been measured for the main mode (\( n_{\ast} = 2.1300 \)).

We have also obtained a unidirectional grating coupler for a combined waveguide
Figure 4. (a) Construction of a unidirectional coupler (inclined beam etching). Curves 1–4 are the resulting shapes of the film–vacuum interface (the calculations are made according to the mathematical model of etching). (b) Radiation fraction into air, calculated curve and experimental points. (c) Unidirectional grating coupler. $P_1, P_2$ the power registered by a photodetector; $\alpha_1/(\alpha_1 + \alpha_2) = P_1/(P_1 + P_2) = 0.97$.

(figure 5 (a)). The waveguide has been fabricated by electrodiffusion of Cs$^+$ ions into a glass substrate ($n_g = 1.512$). The refractive-index profile of this waveguide is close to rectangular, with maximum refractive-index change $n_3 - n_4 = 0.034$, and the waveguide depth $d = 0.72 \mu$m. The waveguide supported a single mode at $\lambda = 0.6328 \mu$m. On the diffused waveguide the diffraction grating was fabricated with the period $\Lambda = 0.321 \mu$m and depth $\sigma_2 = 200 \AA$. The corrugated waveguide was coated with Nb$_2$O$_5$ film of $h = 500 \AA$ (below cut-off), forming a double corrugated layer with $\varphi = 0.7$ rad. The technique of formation was the same as in the previous case. A Nb$_2$O$_5$ film of thickness 900 Å was first sputtered and the sample was then exposed to oblique incident ion-beam etching for pre-calculated time. The device exhibited preferential radiation coupling into the air $\alpha_1/(\alpha_1 + \alpha_2) = 0.80$. As mentioned above, this value does not reach 100% just because the grating depths are equal due to the low initial thickness of Nb$_2$O$_5$ film ($\sigma_1 = \sigma_2$) and are not matched in accordance with (13). A choice of the optimal phase makes it possible to provide predominant radiation coupling into the air, but unidirectional coupling is not still realized.
Interference phenomena in waveguides

\[ \lambda = 0.63 \, \mu m \]

\[ \text{Nb}_2\text{O}_5 \]

\[ \text{Cs}^+ \]

\[ \text{Glass} \]

\[ \text{P}_1 \]

\[ \text{P}_2 \]

(a)

\[ \text{Nb}_2\text{O}_5 \]

\[ \text{Cs}^+ \]

\[ \text{Glass} \]

\[ \text{Glycerine} \]

(b)

Figure 5. (a) Unidirectional coupler based on the combined waveguide; \( \alpha_1/(\alpha_1 + \alpha_2) = P_1/(P_1 + P_2) = 0.80 \). (b) Unidirectional grating coupler based on the combined waveguide with an immersed diffraction grating; \( P_1/P_1 + P_2 = 0.96 \).

For condition (13) to be satisfied, the normalized corrugation depth \( S_1 \) at the film–air interface must be reduced. The possibilities for a variation of the grating depth are limited, so we applied another technique, namely immersion of the diffraction grating (figure 5(b)). The difference of the refractive indices at the film–superstrate interface is thus reduced and the relationship between the normalized grating depths is close to optimal from the point of view of the radiation minimization into the substrate. Thus, using glycerine as the immersion fluid \( (n = 1.47) \), we were able to achieve unidirectional radiation coupling from the combined waveguide \( (\alpha_1/(\alpha_1 + \alpha_2) = 0.96) \). The 4% of the coupled-out radiation that was detected on the substrate side resulted from Fresnel reflection from the glass slide–glycerine interface.

Note also that in a practical application of the unidirectional grating coupler one can use polymer films with a flat film–air interface as immersing media.

5. Anomalous light reflection from a double corrugated waveguide.

**Oblique light incidence**

The corrugated waveguide can exhibit anomalous light reflection [8–10] under excitation with a plane electromagnetic wave. The amplitude of the reflected wave can vary from 0 to 1, and the phase can monotonically change by \( 2\pi \) depending on the excitation condition [11]. Owing to the resonance character of the waveguide excitation, these anomalies are concentrated in rather narrow spectral and angular
intervals

\[ \Delta \lambda \approx \frac{\Lambda \lambda^2}{2\pi}, \]

(17)

\[ \Delta \theta \approx \frac{\lambda^2}{2\pi \cos \theta}, \]

(18)

where \( \theta \) is the angle of light incidence. A phenomenological approach developed in [11] leads to the conclusion that 100% reflection is achieved for the waveguides without dissipative losses when \( \varphi = 0 \) and \( \varphi = \pi \). For a double-corrugated waveguide when \( \varphi \neq 0, \pi \) no conclusions can be drawn from the phenomenological theory.

It is natural to expect that some specific features of anomalous reflection must be observed for the waveguides with unidirectional couplers. We have calculated the reflection spectra for the waveguide with the following parameters: \( n_1 = 1, n_2 = 1.46 \) (SiO₂), \( n_3 = 1.433 \) (CaF₂), \( h = 0.7 \) μm, \( A = 0.37 \) μm, \( \sigma_1 = 0.01 \) μm, \( \sigma_2 = 0.056 \) μm and different values of phase \( \varphi \). The structure, providing a unidirectional radiation coupling, turned out to exhibit no anomalous reflection (curve 6, figure 6). This is quite natural, since anomalous reflection is caused by wave interference, radiated from the waveguide and reflected from its boundaries. An analogous phenomenon must have been observed for the waveguides with asymmetric grating and with the

Figure 6. Anomalous light reflection spectrum from a double corrugated waveguide: \( \varphi = 0, \pi/4, \pi/2, 2\pi, \pi \) and 1.08 rad for curves 1–6 respectively.
bulk dielectric gratings having blazing grooves which provide preferential radiation coupling.

6. **Anomalous reflection. Normal incidence**

The case of normal light incidence is of special interest. It is characterized by the following: at normal incidence two waveguide modes are excited, propagating in opposite directions and interacting in the second diffraction order.

It is known for the case of one-side-corrugated waveguides that, at small incidence angles, two characteristic peaks are available in the reflection spectrum, which are caused by the waveguide excitation in the $+1$ and $-1$ diffraction orders. Their spectral separation decreases with decrease of $\theta$, down to a value proportional to the coupling coefficient of the waveguide modes propagating in the opposite directions. The spectral width of the short wave peak increases approximately two-fold with respect to the non-resonance case ($\theta \neq 0$), and that of the long-wave peak reduces to zero as $\theta \to 0$. Thus, when $\theta = 0$, only one short-wave reflection peak remains in the spectrum of anomalous reflection. The question remains open as to whether such a difference in behaviour of the short- and long-wave reflection peaks is a common feature of the corrugated waveguides, and what is it caused by.

In an attempt to answer these questions, we have calculated the reflection spectra with $n_1 = 1$, $n_2 = 1.46$, $\lambda = 0.056 \mu m$ and $h = 0.41 \mu m$, $A = 0.388 \mu m$, $\sigma_1 = 0.015 \mu m$ and $\sigma_2 = 0.0526 \mu m$ at different $\varphi$ values (figure 7).

A relationship between the grating depths is chosen taking the waveguide asymmetry into account, such that the influence of each depth upon the anomalous reflection spectrum is approximately equal. The angle of incidence was taken to be $\theta = 2 \times 10^{-5}$ rad to ensure that the interaction between modes propagating in the opposite direction was sufficiently high. When $\varphi = 0$ or $\varphi = \pi$ the reflection coefficient in both reflection peaks reaches unity, but when $\varphi \neq 0$, $\pi$ this does not occur. When $\varphi \neq 0$, $\pi$, both the maximum reflection $R_{max}$ (figure 7(d)) and the spectral width of the reflection peaks $\delta \lambda$ depend on the phase $\varphi$. It should be noted that in contrast to the non-resonant case (large angles of incidence), anomalous reflection peaks exist at any $\varphi \neq 0$, $\pi$. This is quite natural, since formulae (8) are derived assuming that the total electromagnetic field is given by (1) and (5), that is, interaction between modes was not taken into account.

The most important result of these calculations is the fact that for an arbitrary phase relationship between the corrugations, two reflection peaks of finite spectral width (not one as for one corrugation) are available in the reflection spectrum at normal incidence ($\theta = 0$). The maximum reflection coefficient for both peaks for the lossless waveguides reaches theoretically 100$. The spectral width $\delta \lambda$ (figure 8) of the long-wave peak at $\varphi = 0$ and $\varphi = \pi$ is equal to zero.

To understand the behaviour of the long- and short-wave reflection peaks, we have calculated the interference pattern of the waveguide modes inside the waveguide at small phase shift $\varphi = \pi/16 \ll \pi$. The value $\varphi$ was chosen such that, on the one hand both peaks of anomalous reflection are available, and on the other they are considerably different with respect to the spectral width. A comparison of the interference patterns shows that for the short wave, peak interference maxima are spaced in the corrugation antinodes, namely in the hollows and tops of corrugation, and for the long-wave peak, in their nodes. When $\varphi \neq 0$, $\pi$ this situation is not principally realized. So, for double-corrugated waveguides with arbitrary phase shift between corrugations, two peaks of finite spectral width are available in the reflection spectrum even at normal incidence.
Figure 7. Anomalous reflection spectra at small angles of incidence, $\theta = 2 \times 10^{-5}$ rad: (a) $\phi = 0$; (b) $\phi = \pi$; (c) $\phi = \pi/4$; and (d) the maximum reflection coefficient as a function of the relative grating phase (I—for a shortwave reflection peak, II—for a longwave reflection peak).

7. Anomalous reflection from the multilayer waveguides

We have already noted that the corrugated waveguide parameters can be varied by deposition different films on the waveguide. In particular, sputtering of a large-refractive-index film enhances the mode field at the waveguide surface, thereby enhancing the grating efficiency. As applied to anomalous reflection, we know that an increase of the waveguide mode coupling coefficient with incident wave leads to
Figure 8. Anomalous reflection at normal incidence. Spectral width of reflection peaks (I—shortwave, II—longwave) against the relative grating phase.

The spectral broadening of the reflection peak. Generally speaking, this is an undesirable effect in the development of frequency filters based on anomalous reflection. The present waveguides, however, have finite dissipative losses. The maximum reflection coefficient is defined by a relationship between radiative $\alpha_r$ and dissipative $\alpha_d$ losses. In particular, for a corrugated diffused waveguide

$$R_{\text{max}} = \left| \frac{r \alpha_d - \alpha_r}{\alpha_d + \alpha_r} \right|^2,$$

where $r$ is the Fresnel reflection from a waveguide without corrugation. With a field increase at the waveguide surface, radiative losses are enhanced. If insertion losses do not grow or grow slightly (for instance if they are caused by a bulk absorption or by a scattering inside the waveguide layer) maximum reflection coefficient increases. This case is illustrated by the calculations of the reflection from Si–SiO$_2$/CaF$_2$ waveguide spectra (figure 9). The waveguide parameters are: $n_1 = 1$, $n_2 = 3.5$, $n_3 = 1.46$, $n_4 = 1.433$, $h = 0.015$ $\mu$m, $d = 1.71$ $\mu$m, $\sigma_1 = \sigma_2 = 0.02$ $\mu$m and $\varphi = 0$. In the presence of dissipative losses ($\alpha_d = 2.5$ $\text{cm}^{-1}$) the Si film on the waveguide enables one to obtain a rather large reflection of 76%, whereas in the absence of the film maximum reflection is not more than 30%.

8. Antireflection coating and anomalous reflection

A non-zero Fresnel reflection far from anomalous reflection peaks should be referred to a disadvantage of the frequency filters based on corrugated waveguides. For glass waveguides, it is not so large (\(\sim 4\%\)) and in the most cases can be neglected. Sometimes the waveguides made of high-refractive-index materials (say,
semiconductors), are required. The spectral selectivity on the basis of corrugated waveguide is defined by a number of the grating grooves on the characteristic waveguide mode coupling length \( l = \alpha^{-1} \) with incident radiation. The grating period of this filter, operating at normal incidence is equal to \( \Lambda = \lambda/n^* \).

Thus using materials with large refractive indices (waveguides with large \( n^* \)), the grating groove density can be increased; spectral selectivity can be realized with the use of small size device.

Fresnel reflection from the semiconductor layers \( (n = 3.5-4.0) \) cannot be neglected and amounts to 30–40%.

The calculations show that antireflecting coating suppresses the Fresnel reflection and practically does not affect the anomalous reflection. Figure 10 illustrates reflection spectra from substrate with the parameters \( n_1 = 1, n_2 = 1.84, n_3 = 3.6 \) (GaAs), \( n_4 = 3.5, d = 0.58 \mu m, h = 0 \) (no antireflecting coating, curve 1) and \( h = 0.18 \mu m \) (curve 2), \( \Lambda = 0.382 \mu m, \sigma_1 = \sigma_2 = 0.01 \mu m \) and \( \varphi = 0 \).

9. Practical application of anomalous light reflection from the corrugated waveguides

Anomalous light reflection can be applied in different kinds of spectral devices: in monochromators (as optical filters), in laser resonators (as dispersion elements) and so on. Their application as the selective elements of laser resonators is the most promising. In the first experiments demonstrating a normal operation of the selective mirrors based on corrugated waveguides, the dye Rhodamine 6G was used as the active element [12, 13]. Use of corrugated waveguides has been found to lead not only to a narrowing of generated spectrum, but to an improvement of the spatial coherence of the radiation. The latter effect is caused by the fact that while reflected from a corrugated waveguide, the beam is widened in anomalous reflection regime, as the waveguide mode carries energy beyond the limits of the radiation spot.
Interference phenomena in waveguides

Figure 10. Anomalous reflection from the corrugated waveguide, with (curve 2) and without (curve 1) antireflecting coating.

A scheme incorporating a tunable laser with a dispersion element based on the corrugated waveguide [12] has, generally speaking, a disadvantage with respect to the scheme with a metal-coated diffraction grating, operating in a Littrow scheme, from the point of view of the dimension of devices. However, if a corrugated waveguide is the output mirror of a laser resonator, one can construct a very compact light source with a stabilized radiation frequency.

The requirement of small size is the main one for semiconductor lasers used in fibre communication lines. So we think that practical interest lies in the study of semiconductor lasers, frequency-stabilized by a selective external cavity mirror, based on corrugated waveguides. In our experiments, we have used an InGaAs semiconductor stripe laser, operating at the wavelength \(1.3 \mu m\). The laser end that faces the corrugated waveguide reflector was coated such that the transmission coefficient was \(\tau = 0.99\). The transmitted radiation was collimated by an objective with aperture \(0.85\) and magnification of \(\times 60\). A selective reflector was placed perpendicular to the collimated light beam. To construct the selective reflector, a diffraction grating with period \(A = 0.842 \mu m\) and full depth \(2\sigma_2 = 0.25 \mu m\) was formed on the polished glass substrate. Then a Ag\(^+\)-diffused waveguide was formed in the substrate surface layer. For this aim, the substrate was inserted for 2.5 min into a silver nitrate (2\%) and sodium nitrate melt at 320°C. Then a thin silicon layer was sputtered \((h = 200 \AA, \sigma_1 = \sigma_2, \varphi = 0)\) on the waveguide surface, and as a result we have obtained a selective reflector with a reflection peak width \(\sim 5 \AA\) and maximum reflection \(R \sim 0.5\) at \(\lambda \sim 1.9 \mu m\).

It was established as a result of investigations that an introduction of the selective mirror external cavity.

1. Reduces the lasing current threshold approximately twofold (from 50 to 25 mA);
2. In ranges of the injection current below the threshold (without external cavity) (25–50 mA), a single longitudinal mode of laser diode is generated. The other modes are 30–35 dB of the main mode intensity;
(3) In the range of injection current 50–80 mA, the external resonator suppresses a wide spectrum of laser radiation, indicating one longitudinal mode of laser diode. The other modes are not more than 15–20 dB of the main mode intensity;

![Diagram of a semiconductor laser with a selective mirror based on a corrugated waveguide.](image)

Figure 11. (a) A schematic diagram of a semiconductor laser with the selective mirror based on a corrugated waveguide. (b) Laser radiation spectrum in different regimes: with an external resonator at a pumping current $I = 40$ mA (curve 1), $70$ mA (curve 2) and $90$ mA (curve 4) and without the resonator at a current $I = 70$ mA (curve 3).
(4) At even higher injection currents (over 90 mA), the radiation is multimode. The mode selection caused by the external resonator turns out to be already ineffective.

The laser radiation spectra oscillograms in different regimes of operation are illustrated in figure 11.

Thus the use of the corrugated waveguides as an external resonator mirror makes it possible to construct a compact narrow-band radiation source based on a semiconductor laser.

10. Conclusions
We have considered corrugated waveguides having two corrugated interfaces with equal periods but different phases and amplitudes. We have also derived the formula for radiative losses in a combined corrugated waveguide (thin film on the graded-index waveguide). Film deposition with a high refractive index is shown to increase the field amplitude at the waveguide surface, and hence the radiative losses of the grating. The relationships are established between the film, grating and waveguide parameters, providing unidirectional radiation coupling into air. Formuлаe are given for radiative losses in corrugated graded-index and double corrugated slab waveguides. Unidirectional radiation coupling from the film and combined waveguide has been experimentally demonstrated. For double-corrugated waveguides with unidirectional coupling, anomalous reflection is shown to be absent in the case of oblique light incidence. For normal incidence and an arbitrary relationship between the corrugation phases, two peaks (not one, as for a one-sided corrugation waveguide) are presented.

It is shown that in structures with finite dissipative losses, the maximum anomalous reflection can be increased by deposition of a high-refractive-index film. It is also shown that after reflection does not disappear and that the spectral characteristics of the frequency filter based on a corrugated waveguide are significantly improved.

The application of the corrugated combined waveguide as a selective element of the external cavity of a semiconductor laser is demonstrated.

References