Nonlinear corrugated waveguide excitation and optical bistability

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Abstract. Optical bistability is considered in the case of nonlinear corrugated waveguide excitation with a plane electromagnetic wave. With some simplifying assumptions it is possible to construct a complete analytical description of this phenomenon. Due to the effect of light anomalous reflection from the corrugated waveguide surface, bistability manifests itself in the reflected and transmitted beams.

1. Introduction

The first experiments on optical bistability were performed in the middle of the 1970s. Since then, numerous and various methods have been developed to induce optical bistability [1], whose essence is reduced to fabricating optical devices with two stable states. The major aim of the research was to reduce the dimensions of the systems and the switching power, and to attain a high-speed action. Among the optical bistable systems most attractive from this viewpoint are those based on waveguides, for instance those using coupled waveguides or nonlinear Bragg diffraction.

We are concerned with optical bistability produced by excitation of a nonlinear corrugated waveguide with a plane electromagnetic wave. In this case the bistability manifests itself in the dependence of reflected and transmitted radiation intensity upon incident radiation intensity. In contrast to other waveguide type devices the incident, transmitted and reflected beams are bulk beams rather than waveguide beams. Thus, the device involved is a very compact waveguide bistable device for bulk optics. Devices of such a type have not been considered sufficiently in the literature.

The possibility of a bistable regime in this case has been demonstrated both theoretically and experimentally [2–4]. The authors used an Nd: YAG pulsed laser and a silicon-on-sapphire waveguide. In [5] the ‘external’ grating coupler is pressed against the waveguide in a similar way to the way a prism coupler is used. The authors used low-loss SiO₂–TiO₂ waveguides and an argon laser. They observed photo-thermal bistability with input powers of 15 mW. Lastly, in [6], an experimental investigation of bistability with a multi-quantum-well (GaAs/GaAlAs) waveguide was described. The bistable behaviour was obtained at an incident power of 8 mW.

In this work we present analytical expressions describing bistability in cases of waveguide excitation and specify the criteria for its existence.
2. Principal assumptions

Let a plane monochromatic wave with an amplitude electromagnetic field value $E_i$ be incident at a corrugated waveguide. In this case the intensity of the electric field of a waveguide mode is proportional to the expression [7]

$$E_w \sim \frac{E_i}{\lambda + n_1 \sin \theta - n^* - \frac{i}{2k} (\alpha_r + \alpha_d)}$$

where $\lambda$ is the light wavelength; $\Lambda$ is the groove period; $n_1$ is the refractive index of the medium in which the incident beam propagates; $\theta$ is the incidence angle; $k = 2\pi/\lambda$; $\alpha_r$ and $\alpha_d$ are radiative and dissipative losses; and $n^*$ is the real part of an effective refractive index with due regard to groove perturbation. The radiation power density in the waveguide is then equal to

$$P = \frac{\beta P_i}{\left(\frac{\lambda}{\Lambda + n_1 \sin \theta - n^*}\right)^2 + (\alpha_r + \delta_d)^2/(2k)^2}$$  \hspace{1cm} (1)

where $P_i = |E_i|^2 n_1$ is the incident wave intensity.

The coefficient $\beta$ characterizes the degree of energy concentration in the waveguide. The value of the coefficient depends on the profile and depth of the diffraction grating, the profile of the waveguide refractive index and the radiation polarization. In the approximation of a small grating depth $\sigma$, parameter $\beta$ as well as radiative losses $\alpha_r$ quadratically increase with increasing $\sigma$, that is

$$\beta = \beta_0 \sigma$$  \hspace{1cm} (2)

and $\beta_0$ does not depend on the grating depth. For the case of TE-polarized radiation, and a sinusoidal grating with a half-depth $\sigma$ and step-like profile of refractive index (figure 1)

$$n(x, z) = \begin{cases} n_1 & z > \sigma \sin Kx \\ n_2 & -h < z < \sigma \sin Kx \\ n_3 & z < -h \end{cases}$$  \hspace{1cm} (3)

we have

$$\beta = \frac{1}{8} \left( \frac{\sigma}{h^*} \right)^2 \left( \frac{n_2^2 - n_1^2}{n_2^2 - n^*} \right) \left| 1 + \rho_F \right|^2$$  \hspace{1cm} (4)

where

$$h^* = h + \frac{1}{k} \left( \frac{1}{\sqrt{(n^* - n_1^2)}} + \frac{1}{\sqrt{(n^* - n_3^2)}} \right)$$

is the effective thickness of the waveguide; $h$ is the waveguide layer thickness; $n_2$ and $n_3$ are the refractive indices of the waveguide layer and substrate, respectively; and $\rho_F$ is the amplitude Fresnel refraction coefficient from the wavelength surface without groove. In this case the average intensity in the layer is determined as the average square of the waveguide mode field in the waveguide layer multiplied by the effective refractive index:

$$P = \frac{n^*}{h} \int_{-h}^{0} |E_w(z)|^2 \, dz.$$  \hspace{1cm} (5)
In the general case, the expression for $\beta_0$ is rather cumbersome, but for a film waveguide, with some additional assumptions made, it has the following form:

$$\beta_0 \approx \frac{2}{k^2 h^*}. \tag{6}$$

For further consideration, equation (1) only is of foremost importance. Relationships (2)–(6) have a particular character and will be needed only for numerical estimation of bistability parameters when the film waveguide is excited.

Another important assumption is that the power increasing in the waveguide changes only the real part of the waveguide effective refractive index and this change is linear:

$$n^* = n_0^* + \gamma P. \tag{7}$$

It should be noted that (7) holds only at relatively small $P$ values or at small values of the nonlinear refraction coefficient. For great nonlinearity, the propagation constant is a rather complex function of $P$ and in some cases it even has a bistable character [8]. With assumptions (1) and (7), the problem of bistability in the excited waveguide is important, since due to the resonance character of excitation, bistability may be expected to occur at far lower power (or nonlinearity coefficient) than in case of radiation propagation in the nonlinear waveguide.

We neglect variation in radiative losses with increasing power caused by the mode profile changes.

In the cases where nonlinear refraction is due, for instance, to generation of free carriers in semiconductors, dissipation losses undergo changes. They grow linearly with increasing power $P$ in the waveguide. This effect can be easily taken into account, which we do below, and we now assume that dissipative losses are the waveguide parameter independent of $P$. Substituting (7) into (1) we have

$$P = \frac{\beta P}{\left(\frac{\Lambda}{\lambda} + n_1 \sin \theta - n_0^* - \gamma P\right)^2 + (\alpha_x + \alpha_0)^2/(2k)^2}. \tag{8}$$
2. Waveguide excitation and bistability

Equation (8) describes bistability occurring on waveguide excitation. It is a third-order equation with respect to $P$ and hence it can be analytically solved with respect to $P$ using the Cardano formulae. However, this cumbersome procedure is not required for determining the principal parameters characterizing bistability on waveguide excitation.

Let us use dimensionless parameters to make further calculations more obvious:

$$
\begin{align*}
    p &= P \gamma (2k/\alpha_s) \\
    p_i &= P_i \beta (2k/\alpha_i) = P_i \gamma (\Delta/2k\alpha_i) (2k/\alpha_i)^2 \\
    \Delta &= \left( \frac{\lambda}{\lambda} + n_1 \sin \theta - n_0^* \right) (2k/\alpha_i) \\
    a &= 1 + \alpha_0/\alpha_s.
\end{align*}
$$

(9)

The physical sense of the $\Delta$ parameter is deviation from the condition of the most effective waveguide excitation at $P \to 0$ normalized to the half-width of dissipationless waveguide excitation (see (1)).

In these terms equation (8) assumes the form

$$
p^3 - 2\Delta p^2 + (\Delta^2 + a^2) p = p_i.
$$

(10)

Let us denote the left-hand side of (10) by $f(p)$. This function is monotonic at $df/dp > 0$, that is at $\Delta^2 \leq 3a^2$, and has extreme values (figure 2) at

$$
\Delta^2 > 3a^2
$$

(11a)

i.e.

$$
\left| \frac{\lambda}{\lambda} + n_1 \sin \theta - n_0^* \right| > \frac{\sqrt{3}}{2k} (\alpha_s + \alpha_0).
$$

(11b)

![Figure 2. Waveguide power, $p$, versus incident wave power, $p_i$.](image)

Only positive values of $P$ have physical meaning, therefore one should consider only such extreme values of function $f(P)$ for which $\gamma P > 0$, resulting in the condition

$$
\Delta/\gamma > 0.
$$

(12)

The values of $p$ for which $df/dp = 0$ are, correspondingly, equal to

$$
p_{1,2} = \frac{1}{3}(2\Delta \pm \sqrt{(\Delta^2 - 3a^2)}).
$$

(13)

Substituting (13) into (10) gives the values of intensity of ‘switching-on’ and ‘switching-off’ of an optical bistable device

$$
\begin{align*}
    P_1 &= \frac{2}{3} [\Delta(\Delta^2 + ga^2) + (\Delta^2 - 3a^2)^{\frac{3}{2}}] \\
    P_i &= \frac{2}{3} [\Delta(\Delta^2 + ga^2) - (\Delta^2 - 3a^2)^{\frac{3}{2}}].
\end{align*}
$$

(14)
The smallest value of the intensity of the incident wave corresponding to switching on of the bistable regime is realized at $\Delta^2 = 3a^2$ and equals

$$P_i^0 = \frac{8a^2}{3\sqrt{3} \beta_0} = \frac{1}{3\sqrt{3} \beta_0 \gamma k^3} (\alpha_0 + \delta_0)^3. \tag{15}$$

As it follows from (15), at fixed dissipative losses for minimization of the incident power, the grating depth has to be fitted such that condition $\alpha = \alpha_d/2$ would hold. In this case

$$P_i^0 = \frac{3\sqrt{3}}{4} \frac{\alpha_0^2}{\beta_0 \gamma k^3} \tag{16a}$$

or, taking (6) into account,

$$P_i^0 = \frac{3\sqrt{3}}{8} \frac{h^* \alpha_0^2}{\gamma k}. \tag{16b}$$

When dissipative losses in the corrugated waveguide are very small, the optimum value of the radiative losses is determined by the size of the incident beam, $l$. In this case, to attain efficient waveguide excitation, one has to provide at the grating at least $\alpha_d \geq l^{-1}$ and, correspondingly, the minimum active power density will be

$$P_i^0 = \frac{1}{3\sqrt{3}} \frac{\alpha_0^2}{\beta_0 \gamma k^3} \geq \frac{1}{3\sqrt{3}} \frac{1}{\beta_0 \gamma k^3} \frac{1}{l^2}. \tag{17a}$$

or, taking (6) into account

$$P_i^0 = \frac{1}{6\sqrt{3}} \frac{h^* \alpha_0^2}{\gamma k} \geq \frac{1}{6\sqrt{3}} \frac{h^*}{\gamma k^2 l}. \tag{17b}$$

For a sufficiently inertialess mechanism of nonlinearity, the triggering time is determined by

$$\tau \simeq n^*/\alpha_c. \tag{18}$$

At the device area $S$ the triggering energy will be [1]

$$Q = P_i \tau S = \frac{1}{6\sqrt{3}} \frac{h^* n^* S \alpha_0^2}{\gamma k}. \tag{19}$$

Thus, when a corrugated nonlinear waveguide is excited by a plane wave, bistability manifests itself in the dependence of radiation intensity $P$ in the waveguide upon incident radiation intensity. In this case, bistability will manifest itself in all diffraction orders including zero-orders (in the transmitted and reflected beams).

3. Bistability in the reflected and transmitted beams

The values of intensity $P$ in the waveguide unambiguously determine the effective refractive index of the waveguide (7). Deviation from the condition of most effective excitation is thus also dependent on the intensity in the waveguide. In the dimensionless units of (9) we have

$$\Delta_\nu = \Delta - p. \tag{20}$$
Figure 5. (a) Bistability in the reflected beam for various shapes of the anomalous reflection line: $R = 0.35; \chi_{o}/\chi = 0; \Delta = 3.0; \varphi = 0 (1), \pi/2 (2), -\pi/2 (3). (b) Reflection line shapes for the same parameters.
Figure 6. Bistability in the reflected (a) and transmitted (b) beams: $R_p = 0.35; \varphi = -\pi/2; \Delta = 3.0; \alpha_0/\alpha = 0 (1), 0.25 (2), 0.5 (3)$.
In the case of film waveguides, the phase of the reflected wave may be changed by multiple reflections of the zero-order waves from the boundaries of the waveguide layer. The reflection line $R(\Delta)$ becomes in this case asymmetric, and $\rho_r(p_r)$ dependence changes significantly. These events offer some opportunities for improving the characteristics of the bistable element. These events are evidently marked at relatively large values of Fresnel reflection. Therefore, the influence of the $\varphi$ phase upon the character of the $\rho_r(r)$ dependence was illustrated for $R_0 = 0.35$ (figure 5). As is seen, the contrast is much better for $\varphi = -\pi/2$. The hysteresis loop width is determined by formulæ (14) and does not depend upon the shape of the reflection line.

In the transmitted beam, bistability is also observed. It has an inversion character with respect to that in the reflected beam. With a dissipative waveguide, bistability in the transmitted beam has, generally speaking, a greater contrast than that in the reflected beam, since minimal transmittance in the mode of anomalous reflection remains very small even for rather large $\alpha_0/\alpha_r$ (figure 6).

4. The influence of nonlinear absorption upon bistability induced by waveguide excitation

Assuming that equations (1) and (7) hold and absorption is linearly dependent upon power in the waveguide, then

$$\alpha_d = \alpha_0^d + \gamma_1 2kP$$

(25)

where $\alpha_0^d$ denotes losses at small power and $\gamma_1$ is some coefficient. Substituting (7) and (25) into (1) gives

$$P = \frac{\beta P_i}{(\Delta - \gamma P)^2 + \left(\frac{1}{2k(\alpha_r + \alpha_0^d) + \gamma_1 P}\right)^2}.$$  

(26)

It is easy to see that (26) is reduced to an equation of the same type as (8),

$$P = \frac{\beta P_i}{\left(\frac{\alpha_r}{2k} \Delta^* - \gamma^* P\right)^2 + a^*}$$

by substituting

$$\Delta^* = [\Delta - (1 + \alpha_0^d/\alpha_r) \gamma_1]/\sqrt{(\gamma^2 + \gamma_1^2)}$$

$$\gamma^* = \sqrt{(\gamma^2 + \gamma_1^2)}$$

$$a^* = [\Delta \gamma_1 + (1 + \alpha_0^d/\alpha_r) \gamma_1]/\sqrt{(\gamma^2 + \gamma_1^2)}.$$  

(27)

The major condition for the presence of bistability, $\Delta^* > 3a^*$, with due regard to the fact that generation of the carriers leads to decreasing refractive index and increasing absorption, i.e. $\gamma < 0$ and $\gamma_1 > 0$, is reduced to the following condition:

$$(1 + \alpha_0^d/\alpha_r) \gamma_1 > \Delta \geq (1 + \alpha_0^d/\alpha_r)(1 - \sqrt{(3/2)}|\gamma_1/\gamma_1|)(1 - \sqrt{(3/2)}|\gamma_1/\gamma_1|)^{-1}.$$  

(28)

As follows from (28), in the case of nonlinear absorption the de-tuning $\Delta$, at which bistability occurs, increases. As a result, this leads to increasing power at which bistability is observed. Moreover, from (12), when

$$\gamma_1 > \Delta \gamma_1/(1 + \alpha_0^d/\alpha_r)$$

(29)
bistability is not available at all. On the other hand, if nonlinear absorption is of a bleaching character \((\gamma_1 < 0)\), then, as follows from (27), (11) and (12), even at \(\gamma = 0\), bistability is realized at

\[ |\Delta| < \sqrt{3}(1 + \frac{\alpha_0}{\alpha_1}). \]

5. Other possible applications of nonlinear corrugated waveguides

Not only bistable optical devices, but other elements of optoelectronics can be derived from anomalous light reflection from the surface of a nonlinear corrugated waveguide. For instance, at \(|\Delta| < \sqrt{3}\alpha\), the \(\rho_1(p_1)\) function is monotonic but has a section with a big slope. Thus, a nonlinear corrugated waveguide can be used to produce an optical

\[ \text{Figure 7. Optical amplification and doubling of the optical sinusoidal envelope frequency on the basis of a nonlinear corrugated waveguide: } (a) \text{ device diagram}; (b) \text{ optical amplifier}; (c) \text{ inversion amplifier}; (d) \text{ optical sinusoidal envelope frequency multiplier.} \]
amplifier. The $p_i(p_f)$ function has a similar section with a negative derivative, i.e. in the transmitted beam it is possible to realize a quadratic transformation of the optical signal envelope, for instance sinusoidal envelope frequency doubling.

For these two devices to work, the pump beam with a constant power $P_i^0$ and the signal beam have to coincide spatially (figure 7).

6. Numerical estimates

As follows from (17), the incident beam power density at which bistability is observed is proportional to $x_i^2$. However, to attain anomalous reflection for a beam with Gaussian distribution, the device area should be $S \geqslant x_i^2$. Therefore, minimum switching power is determined only by the nonlinearity coefficient and waveguide parameters $kh^*$ and $\alpha_d/\alpha_r$.

A quite attainable value is about 100 cm$^{-1}$, correspondingly $\alpha_r/2k \sim 2 \times 10^{-3}$. The $kh^*$ parameter strongly depends upon the waveguide type. For film waveguides with the refractive index changing abruptly at the waveguide layer–substrate boundary, it may be reduced to $(kh^*)_r \sim 4$. For diffusion waveguides with a small increment of the refractive index this parameter is higher by an order of magnitude $(kh^*)_d \sim 40$.

To estimate the incident beam intensity required for bistability, the value of the $\gamma$ coefficient has to be given. In the film waveguide, a large $\gamma$ can be obtained by producing a waveguide from semiconductors and incorporating inside the waveguide layer a structure of the superlattice type. In such a structure the coefficient of nonlinear refraction reaches $\eta^{21} \sim 0.2$ cm$^2$ kW$^{-1}$ [11]. Such great nonlinearity is, however, observed in the region of excitation resonances featuring high absorption, about $10^4$ cm$^{-1}$. Dissipative losses in the waveguide will be acceptably small if the waveguide mode profile only partially involves the superlattice [12]. In this case $\gamma$ will decrease to the same degree, i.e. having reduced $\alpha_d$ to a value, say, of 100 cm$^{-1}$, we obtain $\gamma \sim 2 \times 10^{-3}$ cm$^2$ kW$^{-1}$. The dimensional unit of intensity will correspond to the value $\sim 2$ W cm$^{-2}$. For a device with an area of $10^4$ mm$^2$ we obtain a power of 200 $\mu$W.

For diffusion waveguides we obtain much greater values of power. If a waveguide is fabricated from a glass with GaS microcrystals, the value of $\gamma = 10^{-7}$ cm$^2$ kW$^{-1}$ can be obtained. The dimensional unit of intensity will be $\sim 10^4$ W cm$^{-2}$.

7. Conclusions

In order to attain bistability by waveguide excitation, the following conditions have to be satisfied. De-tuning from the condition of the most effective waveguide excitation at small incident power $\Delta$ should have the same sign as the coefficient of nonlinear refraction (12). The value of the de-tuning at which the bistable regime is possible is determined only by total losses in the waveguide (11) and by the ratio between the coefficients of nonlinear refraction and nonlinear absorption (28). The power range in which the bistable regime is attainable increases with de-tuning (14) and is inversely proportional to the nonlinear refraction coefficient. The minimal power at which the bistable regime is possible can be attained in waveguides with small dissipation and is determined by relationship (17).

The nonlinear corrugated waveguide operating in the mode of anomalous reflection can serve as a basis for developing an array of optoelectronic elements, such as bistable optical elements, inversion and non-inversion amplifiers and optical sinusoidal envelope frequency doublers.
References