Lecture 12

Analysis of Quiz 1
Fall 2007 ECE 5870 Quiz 1
Open books, open notes

1. (2pt) The Raman gain in silicon is estimated to be $10^2$-$10^4$ times higher than in fused silica, which makes it possible to fabricate compact integrated-optical Raman amplifiers. Assuming the Stokes shift of 15.6THz and FWHM of 96GHz, find the pump wavelength to achieve amplification at 1550nm. Find the wavelength range (FWHM) covered by the amplifier.

2. (2pt) In a graded-index plastic multimode fiber with cladding index $n = 1.55$, the condition for total internal reflection is satisfied for rays propagating at an angle up to $10^\circ$ with respect to the fiber axis. Find the inter-modal dispersion of the fiber and estimate the bit rate-length product. Assume the tolerance factor to be $\varepsilon = \frac{1}{2}$.

3. (2pt) A chirp Gaussian pulse is launched into a fiber with strong negative dispersion ($D < 0$, thus $\beta_2 > 0$). Using the taps installed close to the beginning of the fiber ($L_0 = 0$) and at distances $L_1 = 1.5$ km and $L_2 = 2.5$ km, the pulse-width is measured to be $T_0 = 20$ ps, $T_1 = 15$ ps, and $T_2 = 28.3$ ps. How long fiber would be needed to produce the shortest possible pulse? Find the smallest possible pulse width.

4. (2pt) A Fabry-Perot filter with negligible internal losses and with resonant frequency that coincides with one of the ITU grid frequencies, provides 40dB suppression for the closest ITU channel 100GHz apart from the resonance. Assuming the free spectral range is much larger than the separation between the ITU frequencies, estimate the filter transmission for the frequencies 50GHz apart from the resonance.

5. (2pt) A receiver provides bit error rate of $BER = 10^{-9}$. Assume that the bit errors are mainly caused by the thermal noise of the photodetector. Find the power penalty (the increase in the received power required to keep the error rate constant) associated with working temperature of the device increased by 20°C.
\[ \lambda_s = 1550 \text{ nm} \quad \frac{f_s}{\lambda} = \frac{3 \times 10^{17} \text{ nm/s}}{1550 \text{ nm}} = 193.5 \text{ THz} \]

\[ \Delta f = 15.6 \text{ THz} \]

\[ f_p = f_s + \Delta f = 208.1 \text{ THz} \]

\[ \lambda_p = \frac{C}{f_p} = \frac{3 \times 10^{17} \text{ nm/s}}{208.1 \times 10^{12} \text{ s}} = 1435 \text{ nm} \]

\[ f = \frac{C}{\lambda} \]

\[ \Delta f_{\text{FWHM}} = \frac{C}{\lambda} \Delta \lambda_{\text{FWHM}} \]

\[ \Delta \lambda_{\text{FWHM}} = \frac{\lambda^2}{C} \Delta f_{\text{FWHM}} \]

\[ \Delta \lambda_{\text{FWHM}} = \frac{(1550 \text{ nm})^2}{3 \times 10^{17} \text{ nm/s}} \cdot 96 \times 10^9 \text{ s} = 0.77 \text{ nm} \]
\( n_2 = 1.55 \)

\( \theta_1 = 10^\circ \)

\( n_1 = ? \)

\[ n_1 \cdot \sin(\theta(x)) = \text{const} \]

\[ n_1 \cdot \sin(90^\circ - 10^\circ) = n_2 \]

\[ n_1 = \frac{n_2}{\sin(90^\circ - 10^\circ)} = \frac{n_2}{\cos(10^\circ)} = \frac{n_2}{0.985} = 1.574 \]

\[ \Delta = \frac{n_1 - n_2}{n_1} = \frac{1.574 - 1.55}{1.574} = 1.5 \cdot 10^{-2} \]

**Modal dispersion**

\[ D_M = \frac{1}{C} \frac{n_1}{8} \Delta^2 = \frac{1.574 \cdot (1.5 \cdot 10^{-2})^2}{8 \cdot 3 \cdot 10^5 \text{km/s}} = 1.48 \cdot 10^{-10} \frac{\text{s}}{\text{km}} = 0.048 \frac{\text{ns}}{\text{km}} \]

\[ BLOD_M < \varepsilon \]

\[ (BL)_{\text{max}} = \frac{1}{2} \frac{1}{D_M} = \frac{1}{2} \frac{1}{0.048} \frac{\text{km}}{\text{ns}} = 338 \frac{\text{Gb}}{\text{s}} \text{ km} \]
\[ T(L) = T_0 \sqrt{\left(1 + \frac{K \beta_2 L}{T_0^2}\right)^2 + \left(\frac{\beta_2 L}{T_0^2}\right)^2} \]

\[ T^2 = T_0^2 \left(1 + 2K \beta_2 \frac{L}{T_0^2} + \left(K^2 + 1\right) \frac{\beta_2^2 L^2}{T_0^4}\right) \]

\[ L = L_1 - 2K \beta_2 L_1 + \left(K^2 + 1\right) \frac{L_1^2}{T_0^2} = T_1^2 - T_0^2 \]

\[ L = L_2 - 2K \beta_2 L_2 + \left(K^2 + 1\right) \frac{L_2^2}{T_0^2} = T_2^2 - T_0^2 \]

\[ (K^2 + 1) \beta_2 \frac{L^2}{T_0^2} = \frac{(T_1^2 - T_0^2) L_2 - (T_2^2 - T_0^2) L_1}{L_1^2 L_2 - L_2^2 L_1} \]

\[ K \beta_2 = \frac{(T_1^2 - T_0^2) L_2^2 - (T_2^2 - T_0^2) L_1^2}{2 \left(L_1 L_2^2 - L_2 L_1^2\right)} \]

\[ \beta_2 = \sqrt{\frac{T_0^2 \left(T_1^2 - T_0^2\right) L_2 - \left(T_2^2 - T_0^2\right) L_1}{L_1 L_2 \left(L_1 - L_2\right) - \frac{\left(T_1^2 - T_0^2\right) L_2^2 - \left(T_2^2 - T_0^2\right) L_1^2}{2 L_1 L_2 \left(L_2 - L_1\right)}}} = 200 \frac{\text{ps}^2}{\text{km}} \]

\[ K = \frac{1}{\beta_2} \frac{(T_1^2 - T_0^2) L_2^2 - (T_2^2 - T_0^2) L_1^2}{2 \left(L_1 L_2^2 - L_2 L_1^2\right)} = -1.33 \]

\[ \min T(L) \quad \frac{d}{dL} \left(1 + 2K \beta_2 \frac{L}{T_0^2} + \left(K^2 + 1\right) \frac{\beta_2^2 L^2}{T_0^4}\right) = 0 \]

\[ 2K \beta_2 \frac{1}{T_0^2} + 2 \left(K^2 + 1\right) \frac{\beta_2^2 L}{T_0^4} = 0 \]

\[ L_{\min} = \frac{-K}{K^2 + 1} \frac{T_0^2}{\beta_2} = 0.961 \text{ km} \]

\[ T(L_{\min}) = T_0 \frac{1}{\sqrt{K^2 + 1}} = 12 \text{ ps} \]
\[ T(f_M) = \frac{1}{1 + \left(\frac{f - f_M}{\text{FWHM}/2}\right)^2} \]

\[ T(f_{M+1}) = 10^{-\frac{40}{10}} = 10^{-4} \]

\[ T(f_{M+\frac{1}{2}\Delta f}) = 4 \cdot 10^{-4} \]

\[ \Delta f = \sqrt{10^4 - 1} = \sqrt{9999} \]

\[ \frac{\Delta f}{\text{FWHM}/2} = 4.999 \]

In logarithmic measure

\[ T_{dB} = 10 \log_{10} (4 \cdot 10^{-4}) = -34 \text{ dB} \]
BER = 10^{-9}

\[ BER = Q \left( \frac{I_1 - I_0}{\sigma_1 + \sigma_0} \right) = Q \left( \frac{I_1}{2 \sigma_{th}} \right) \]

\[ x \approx 6 \quad \text{for} \quad \text{BER}=10^{-9} \]

To keep BER constant

\[ \frac{I_1}{2 \sigma_{th}} = x = \text{const} \quad I_1 = 2 \times 6 \sigma_{th} \]

At 300\textdegree\,K

\[ I_1(300\textdegree\,K) = 2 \times 6 \sigma_{th}(300\textdegree\,K) \]

At 320\textdegree\,K

\[ I_1(320\textdegree\,K) = 2 \times 6 \sigma_{th}(320\textdegree\,K) \]

\[ \frac{I_1(320\textdegree\,K)}{I_1(300\textdegree\,K)} = \frac{6 \sigma_{th}(320\textdegree\,K)}{6 \sigma_{th}(300\textdegree\,K)} = \frac{\sqrt{4 k_B T_2 \beta e}}{\sqrt{4 k_B T_1 \beta e}} = \sqrt{\frac{320}{300}} \text{ K} \]

\[ \frac{I_1(320\textdegree\,K)}{I_1(300\textdegree\,K)} = \sqrt{\frac{320}{300}} = 1.033 \]

\[ 10 \log (1.033) = 0.14 \text{ dB} \]