Lecture 13
MidTerm Exam
1. Set theory (6 pt).
Given sets A, B, and C such that \((A \subseteq B) \land (A \cap B \cap C) = \emptyset\), is it always correct that \((A \cap C) = \emptyset\). Draw an illustration.

2. Relations and Mappings (6pt).
Consider a binary relation \(B \subseteq \mathbb{R} \times \mathbb{R}\) such that \(((x, y) \in B) \iff (x \in \mathbb{R}) \land (y \in \mathbb{R}) \land (y = x|x|/(1 + x^2))\). Is this a mapping \((x \rightarrow y)\)? If yes, define the domain, codomain, range, and find out if it is an injection and a surjection.

Find the truth value of the formula
\[
((A \rightarrow B) \land ((A \land B) \land C) \iff O)) \rightarrow ((A \land C) \iff O)
\]
where \(O\) is nonspecific contradiction: \(t(O) = 0\).

4. Algebraic Structures (6pt).
In a set \(S\) of real solutions of the differential equation \(d^4y/dx^4 = -16y(x)\) consider a subset \(Q\) of functions such that \(y(0) = 0\). Is \(Q\) a linear subspace? If yes, find its dimension and basis.

5. Linear Mappings and Matrices (6pt).
Present matrix
\[
\begin{pmatrix}
2 & 4 \\
8 & 6
\end{pmatrix}
\]
in the diagonal form and find \(\hat{A}^{10}\).