Lecture 26
Quiz#2
1. (3 pt). In a real 2-dimensional Hilbert space with vectors \( u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \) the inner-product-induced norm appears to be

\[
\|u\| = \sqrt{10u_1^2 + 5u_2^2 - 2u_1u_2}.
\]

Find the inner product that generates this norm:

\[
\langle u, v \rangle = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = ?
\]

With this inner product, give an example of a vector that is orthogonal to the vector \( u = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \).

2. (3 pt) Is the following inequality always true for any vectors \( a, b, c \) and \( d \) in a Banach space?

\[
\frac{\|a - c\|_2}{2} + \frac{\|b - d\|_2}{2} + \frac{\|c - b\|_2}{2} + \frac{\|d - a\|_2}{2} \leq \|a\| + \|b\| + \|c\| + \|d\|.
\]

3. (3 pt) A linear space is built as a span of vectors

\[
v_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}.
\]

Find the dimensionality of the space and build an orthonormal basis.

4. (3 pt) Find inverse Fourier transform of \( F(\nu) = \nu \cdot \exp(-2\pi |\nu|) \)

5. (3 pt) Classify the following differential equation and transform it to the canonical form.

\[
10 \frac{\partial^2 f}{\partial x^2} + 5 \frac{\partial^2 f}{\partial x \partial y} + 10 \frac{\partial^2 f}{\partial y^2} + 5 \frac{\partial f}{\partial x} + 5 \frac{\partial f}{\partial y} + f = 0
\]