Robust Hovering Control of a PVTOL Aircraft

Feng Lin, *Member, IEEE,* William Zhang, and Robert D. Brandt

***Abstract*—We study robust hovering control of vertical/short takeoff and landing (V/STOL) aircraft. For hovering control, we can model a V/STOL aircraft as a planar vertical takeoff and landing (PVTOL) aircraft. We use a recently developed optimal control approach to design a robust hovering control. The resulting control is a nonlinear state feedback whose robustness is demonstrated by numerical simulations.**

***Index Terms*—Aircraft, hovering control, nonminimum phase, optimal control, Riccati equation, robust control, V/STOL aircraft.**

I. INTRODUCTION

A VERTICAL/SHORT takeoff and landing (V/STOL) aircraft, such as the Harrier (YAV-8B) produced by McDonnell Douglas [14], is a highly maneuverable jet aircraft. The Harrier is powered by a single turbo-fan engine with four exhaust nozzles which provide the gross thrust for the aircraft. These nozzles (two on each side of the fuselage) are mechanically slaved and have to rotate together. They can move from the aft position forward approximately 100° allowing jet-borne flight and nozzle braking. Therefore, the Harrier has two modes of operations, in addition to the transition between the two modes.

1) Wing-borne forward flight as a fixed-wing jet aircraft: In this mode of flight, the four exhaust nozzles are in the aft position. The control is executed by the conventional aerodynamic control surfaces: aileron, stabilator (for *stabil*izer-eleva*tor*), and rudder for roll, pitch, and yaw moments, respectively. (For a survey of wing-borne flight control, see [5].)
2) Jet-borne maneuvering (hovering): In this mode, the four exhaust nozzles are in the forward position, allowing the thrust to be directed vertically. In addition to the throttle and nozzle controls, the Harrier also utilizes another set of controls (reaction control valves) to provide moment generation. Reaction control valves (called puffers) in the nose, tail, and wingtips use bleed air from the high-pressure compressor of the engine to produce thrust at these points and therefore moments (and forces) at the aircraft center of mass. Lateral motion control is accomplished through roll attitude control (rolling moment). It is this mode of flight that we concentrate on in this paper.

We use an optimal control approach [6]–[13] to design the robust control law for the Harrier jet-borne maneuver. The resulting control law has excellent performance as demonstrated by simulations in Section V.

We will first present the optimal control approach in the next section. We will then model the aircraft in Section III and design a robust control in Section IV.

II. PRELIMINARY

We first present a result obtained in [12] that serves as the theoretical base of our approach. We consider the system given by

$$\dot{x} = A(x) + B(x)u + C(x)D(x)u$$

where $D(x)$ is the uncertainty and $C(x) \neq B(x)$ so the matching condition is not assumed. We will assume that $x = 0$ is an equilibrium ($A(0) = 0$). We would like to solve the following.

Robust Control Problem: Find a feedback control law $u = u_0(x)$ such that the closed-loop system

$$\dot{x} = A(x) + B(x)u_0(x) + C(x)D(x)u_0(x)$$

is globally asymptotically stable for all uncertainties $D(x)$ such that $\|D(x)\| \leq D_{\max}(x)$ for some $D_{\max}(x)$.

Since u will be a function of x: $u = u_0(x)$, we can view $f(x) = D(x)u_0(x)$ as uncertainty and guess its bound

$$\|f(x)\| \leq g_{\max}(x),$$
$$\|B(x)^+C(x)f(x)\| \leq \|B(x)^+C(x)\|g_{\max}(x)$$

where $^+$ denotes the (Moore–Penrose) pseudoinverse.

Instead of solving the above robust control problem directly, we will solve the following.

Optimal Control Problem: For the auxiliary system

$$\dot{x} = A(x) + B(x)u + (I - B(x)B(x)^+)C(x)v$$

find a feedback control law $(u_0(x), v_0(x))$ that minimizes the cost functional

$$\int_0^\infty ((\|B(x)^+C(x)\|^2 + \rho^2)g_{\max}(x)^2 + \beta^2\|x\|^2 + \|u\|^2 + \rho^2\|v\|^2)\, dt$$

where ρ and β are some (positive) constants that serve as design parameters.

The following theorem is proved in [12], which shows that we can solve the robust control problem by solving the optimal control problem.

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F. Lin is with the Department of Electrical and Computer Engineering, Wayne State University, Detroit, MI 48202 USA.

W. Zhang is with IBM, Southfield, MI 48078 USA.

R. D. Brandt is with the Computer Systems Engineering Department, University of Arkansas, Fayetteville, AR 72701 USA.

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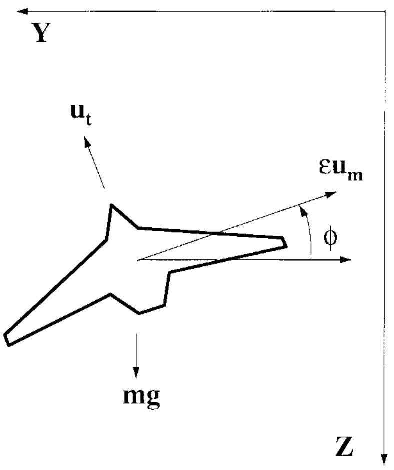

Fig. 1. PVTOL aircraft.

Theorem 1: If one can choose ρ, β, and $g_{\max}(x)$ such that the solution to the optimal control problem, denoted by $(u_0(x), v_0(x))$, exists and the following conditions are satisfied:

$$2\rho^2\|v_0(x)\|^2 \le \beta'^2\|x\|^2,$$
$$\|D(x)u_0(x)\|^2 \le g_{\max}(x)^2, \qquad \forall x \in \mathbb{R}^n$$

for some β' such that $|\beta'| < |\beta|$, then $u_0(x)$, the u-component of the solution to the optimal control problem, is a solution to the robust control problem. ■

We will apply this theorem to design a robust hovering control for V/STOL aircraft.

III. MODELING AND FORMULATION

Since we are interested in control of jet-borne maneuver (hovering), we consider a prototype planar vertical takeoff and landing (PVTOL) aircraft. This system is the natural restriction of a V/STOL aircraft to jet-borne maneuver in a vertical-lateral plane. This prototype PVTOL aircraft as shown in Fig. 1 has a minimum number of states and inputs but retains many of the features that must be considered when designing control laws for a real aircraft such as the Harrier. The aircraft state is simply the position, Y, Z, of the aircraft center of mass, the roll angle, ϕ, of the aircraft, and the corresponding velocities, $\dot{Y}$, $\dot{Z}$, $\dot{\phi}$. The control inputs, U_t and U_m are, respectively, the thrust (directed out the bottom of the aircraft) and the rolling moment about the aircraft center of mass.

In the Harrier, the roll moment reaction jets in the wingtips create a force that is not perpendicular to the Y-body axis. Thus, the production of a positive rolling moment (to the pilot's left) will also produce a slight acceleration of the aircraft to the right. As we will see, this phenomenon makes the aircraft nonminimum phase. Let $\epsilon_0 > 0$ be the small coefficient giving the coupling between the rolling moment and the lateral force, $\epsilon_0 U_m$, on the aircraft. Note that $\epsilon_0 > 0$ means that applying a (positive) moment to roll to the pilot's left produces an acceleration, $\epsilon_0 U_m$, to the right.

In the modeling of the PVTOL aircraft, we neglect any flexure effect in the aircraft wings or fuselage and consider the aircraft as a rigid body. From Fig. 1, we can have the following dynamic model of the PVTOL aircraft:

$$-m\ddot{Y} = -U_t \sin\phi + \epsilon_0 U_m \cos\phi \tag{1}$$
$$-m\ddot{Z} = U_t \cos\phi + \epsilon_0 U_m \sin\phi - mg \tag{2}$$
$$J\ddot{\phi} = U_m \tag{3}$$

where mg stands for the gravitational force exerted on the aircraft center of mass and J is the mass moment of inertia about the axis through the aircraft center of mass and along the fuselage.

For simplicity, we scale this model by dividing (1) and (2) by mg, and (3) by J, to obtain

$$\frac{d^2}{dt^2}\begin{bmatrix} -Y/g \\ -Z/g \end{bmatrix} = \begin{bmatrix} -\sin\phi & \cos\phi \\ \cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} U_t/(mg) \\ \dfrac{\epsilon_0 J}{mg} U_m/J \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$
$$\ddot{\phi} = U_m/J.$$

Let us define

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -Y/g \\ -Z/g \end{bmatrix}$$
$$\begin{bmatrix} u_t \\ u_m \end{bmatrix} = \begin{bmatrix} U_t/(mg) \\ U_m/J \end{bmatrix}. \tag{4}$$

In addition, from now on, we replace $\epsilon_0 J/(mg)$ by ϵ. Then the rescaled dynamics becomes

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -\sin\phi & \cos\phi \\ \cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} u_t \\ \epsilon u_m \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$
$$\ddot{\phi} = u_m. \tag{5}$$

Obviously, at steady state, $u_t = U_t/(mg) = 1$, i.e., the thrust should support the aircraft weight to keep it steady.

Next, we analyze the internal stability of system (5) by looking at its zero dynamics. The zero dynamics of a nonlinear system are the internal dynamics of the system subject to the constraint that the outputs (and, therefore, all derivatives of the outputs) are set to zero for all time.[1] For our PVTOL system, the outputs are the position of the aircraft center of mass, x and y, and the internal state is the rolling angle ϕ and its derivative $\dot{\phi}$.

In (5), the matrix operating on the controls is nonsingular (barely—its determinant is $-\epsilon$!). Therefore, for $\epsilon \neq 0$, constraining the output (x, y) and their derivatives to zero results in

$$\begin{bmatrix} u_t \\ u_m \end{bmatrix} = \begin{bmatrix} \cos\phi \\ \sin\phi/\epsilon \end{bmatrix}.$$

Then, the zero dynamics of (5) are given by

$$\ddot{\phi} = \frac{\sin\phi}{\epsilon}. \tag{6}$$

Equation (6) is simply the equation of an undamped pendulum. It has two sequences of equilibria. One sequence is unstable and the other is stable but not asymptotically stable.

Nonlinear systems, such as (5), with zero dynamics that are not asymptotically stable are called nonminimum phase.

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[1] For detailed discussions on zero dynamics, see [1] and [3].

Based upon this fact, it was shown in [2] that the tracking control designed through exact input–output linearization of the PVTOL system (5) can produce undesirable results (periodic rolling back and forth and unacceptable control law). The source of the problem lies in trying to control modes of the system using inputs that are weakly (ϵ) coupled rather than controlling the system in the way it was designed to be controlled. As pointed out in [2], for the PVTOL aircraft, we should control the linear acceleration by vectoring the thrust vector (using the rolling moment, U_m, to control this vectoring) and adjusting thrust magnitude using the throttle (U_t).

Based on the above discussion, we formulate the robust control problem as follows: to design the control law which can accomplish the jet-borne lateral motion (hovering), say from $x = 1$, or -1 to $x = 0$. This control law has to be robust with respect to the variation of the coupling parameter ϵ.

From the practical point of view, any acceptable control design should satisfy the following requirements.

1) The PVTOL aircraft altitude $y(t)$, in the hovering mode, should have very small deviation from the prespecified altitude, say $y = 0$. Vertical takeoff and landing aircraft are designed to be maneuvered in close proximity to the ground. Therefore it is desirable to find a control law that provides exact tracking of altitude if possible.
2) $u_t \geq 0$, because $U_t = mgu_t$ is the thrust directed out to the bottom of the aircraft. Vectoring of the thrust is accomplished through the rolling moment U_m.
3) $|\phi| \ll \pi/2$, because most V/STOL aircraft do not have a large enough "thrust to weight ratio" to maintain level flight with a large roll angle ϕ.
4) Large control inputs are not acceptable because of the limitations on the maximum thrust and rolling moment generated by bleed air from the high-pressure compressor of the engine.

Any control law which violates one of the above four requirements should be rejected. In the next section, we will seek a robust control law which satisfies the above requirements using the optimal control approach.

IV. CONTROL DESIGN FOR JET-BORNE HOVERING

As the first step toward the robust control design for jet borne hovering of the PVTOL aircraft, we make the following control substitution, which is obviously one-to-one:

$$\begin{bmatrix} u_t \\ u_m \end{bmatrix} = \begin{bmatrix} 1 & -\epsilon \tan\phi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 + u_1 \\ u_2 \end{bmatrix}. \tag{7}$$

Hence, the scaled model (5) becomes

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -\sin\phi \\ \cos\phi - 1 \end{bmatrix} + \begin{bmatrix} -\sin\phi & 0 \\ \cos\phi & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \dfrac{\epsilon}{\cos\phi} \\ 0 \end{bmatrix} u_2 \tag{8}$$

$$\ddot{\phi} = u_2. \tag{9}$$

The objective of making the above control substitution is of two-fold: 1) to make the aircraft altitude $y(t)$ independent of ϵ and hence independent of the lateral force generated by the rolling moment u_2. It is required that the aircraft altitude y has very small deviation from the desired altitude. Through this substitution, y is no longer directly perturbed by ϵ and 2) to make the velocity vector $(\dot{x}, \dot{y}, \dot{\phi})$, acceleration vector $(\ddot{x}, \ddot{y}, \ddot{\phi})$, and the new control (u_1, u_2) go to zero at steady state.

For convenience, we introduce the six-dimensional state vector

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

with $x_1 = [x, y, \phi]^T$ and $x_2 = [\dot{x}, \dot{y}, \dot{\phi}]^T$. Furthermore, we define the following matrices:

$$A(\phi) = \begin{bmatrix} -\sin\phi \\ \cos\phi - 1 \\ 0 \end{bmatrix};$$

$$B(\phi) = \begin{bmatrix} -\sin\phi & 0 \\ \cos\phi & 0 \\ 0 & 1 \end{bmatrix};$$

$$C(\phi) = \begin{bmatrix} \dfrac{1}{\cos\phi} \\ 0 \\ 0 \end{bmatrix}.$$

Then (8) and (9) can be written as

$$\ddot{x}_1 = A(\phi) + B(\phi) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + C(\phi)\epsilon u_2$$

or, equivalently,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ A(\phi) \end{bmatrix} + \begin{bmatrix} 0 \\ B(\phi) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ C(\phi) \end{bmatrix} \epsilon u_2. \tag{10}$$

This is the type of system studied in Section II. So we can now use Theorem 1 to solve this robust control problem. We view $f(\mathbf{x}) = \epsilon u_2(\mathbf{x})$ as uncertainty and guess $\|f(\mathbf{x})\| \leq k\|\mathbf{x}\|$ for some $k > 0$. That is, we take $g_{\max}(\mathbf{x}) = k\|\mathbf{x}\|$. To obtain the corresponding optimal control problem, we define

$$\tilde{B} = \begin{bmatrix} 0 \\ B(\phi) \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} 0 \\ C(\phi) \end{bmatrix}.$$

Since

$$\tilde{B}^T \tilde{B} = B(\phi)^T B(\phi) = I_{2\times 2}$$

we immediately know

$$\begin{aligned} \tilde{B}^+ &= (\tilde{B}^T \tilde{B})^{-1} \tilde{B}^T = \tilde{B}^T \\ B(\phi)^+ &= (B(\phi)^T B(\phi))^{-1} B(\phi)^T = B(\phi)^T. \end{aligned}$$

Therefore

$$(I_{6\times 6} - \tilde{B}\tilde{B}^+)\tilde{C} = \begin{bmatrix} 0 \\ (I_{3\times 3} - B(\phi)B(\phi)^T)C(\phi) \end{bmatrix}$$

with

$$(I_{3\times 3} - B(\phi)B(\phi)^T)C(\phi) = \begin{bmatrix} \cos\phi \\ \sin\phi \\ 0 \end{bmatrix}.$$

Hence, the dynamics of the optimal control problem are

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix}x_2\\ A(\phi)\end{bmatrix} + \begin{bmatrix}0\\ B(\phi)\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} + \begin{bmatrix}0\\ (I-B(\phi)B(\phi)^T)C(\phi)\end{bmatrix}v\\ &= \begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\phi}\\ -\sin\phi\\ \cos\phi-1\\ 0\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ -\sin\phi & \cos\phi & 0\\ \cos\phi & \sin\phi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ v\\ u_2\end{bmatrix}.\end{aligned}$$

or

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_2\\ A(\phi)\end{bmatrix} + \begin{bmatrix}0\\ T(\phi)\end{bmatrix}\begin{bmatrix}u_1\\ v\\ u_2\end{bmatrix} \tag{11}$$

where

$$T(\phi) = \begin{bmatrix}-\sin\phi & \cos\phi & 0\\ \cos\phi & \sin\phi & 0\\ 0 & 0 & 1\end{bmatrix}.$$

It is not difficult to see the following.

Proposition 1: The matrix $T(\phi)$ is symmetric and orthogonal, i.e.,

$$T(\phi)^T = T^{-1}(\phi) = T(\phi).$$

Proof: Elementary. ■

To derive the cost function for the optimal control problem, let us recall the requirement that $|\phi| \ll \pi/2$. In other words, we can find $0 < \phi_0 < \pi/2$ such that $|\phi| \le \phi_0$. Therefore

$$\|\tilde{B}^+\tilde{C}\|^2 = \|B(\phi)^T C(\phi)\|^2 \le \tan^2\phi_0.$$

Hence, the cost functional of the optimal control problem is of the form

$$\begin{aligned}&\int_0^\infty ((\|\tilde{B}^+\tilde{C}\|^2 + \rho^2)g_{\max}^2 + \beta^2\|\mathbf{x}\|^2 + \|u\|^2 + \rho^2\|v\|^2)\,dt\\ &\quad= \int_0^\infty ((\tan^2\phi_0 k^2 + \rho^2 k^2 + \beta^2)(\|x_1\|^2 + \|x_2\|^2) + u_1^2\\ &\qquad + u_2^2 + \rho^2 v^2)\,dt.\end{aligned} \tag{12}$$

In order to be able to solve this (nonlinear) optimal control problem analytically, we take $\rho = 1$. Note that when $\rho = 1$, $\tan^2\phi_0 k^2 + \rho^2 k^2 = \tan^2\phi_0 k^2 + k^2 = k^2/\cos^2\phi_0$.

By the Hamilton–Jacobi–Bellman equation

$$\min_{u_1, v, u_2}\left((k^2/\cos^2\phi_0 + \beta^2)(\|x_1\|^2 + \|x_2\|^2) + u_1^2 + v^2 + u_2^2 + V_x^T\left(\begin{bmatrix}x_2\\ A(\phi)\end{bmatrix} + \begin{bmatrix}0\\ T(\phi)\end{bmatrix}\begin{bmatrix}u_1\\ v\\ u_2\end{bmatrix}\right)\right) = 0$$

we know, if we let (u_{10}, v_0, u_{20}) be the optimal solution and denote $w^2 = k^2/\cos^2\phi_0 + \beta^2$, then

$$w^2(\|x_1\|^2 + \|x_2\|^2) + \left\|\begin{bmatrix}u_{10}\\ v_0\\ u_{20}\end{bmatrix}\right\|^2 + V_x^T\left(\begin{bmatrix}x_2\\ A(\phi)\end{bmatrix} + \begin{bmatrix}0\\ T(\phi)\end{bmatrix}\begin{bmatrix}u_{10}\\ v_0\\ u_{20}\end{bmatrix}\right) = 0. \tag{13}$$

Let us now assume that ϕ will vary in a small neighborhood of zero, so that we can linearize $A(\phi)$ around 0

$$A(\phi) \approx \begin{bmatrix}0 & 0 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ \phi\end{bmatrix} = A_0 x_1$$

and hence

$$\begin{bmatrix}x_2\\ A(\phi)\end{bmatrix} \approx \begin{bmatrix}0 & I\\ A_0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}. \tag{14}$$

Notice that this is the only approximation we make in the process of solving the optimal control problem.

Taking the partial of (13) with respect to the control vector gives

$$\begin{bmatrix}u_{10}\\ v_0\\ u_{20}\end{bmatrix} = -\tfrac{1}{2}\,[0,\, T(\phi)]V_x. \tag{15}$$

Through substituting (14) and (15) into (13) and using Proposition 1, the Hamilton–Jacobi–Bellman equation reduces to

$$w^2(\|x_1\|^2 + \|x_2\|^2) - \tfrac{1}{4}V_x^T\begin{bmatrix}0 & 0\\ 0 & I\end{bmatrix}V_x + V_x^T\begin{bmatrix}0 & I\\ A_0 & 0\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\end{bmatrix} = 0.$$

In order to solve for V from the above equation, we make the guess that V is of the form

$$V(\mathbf{x}) = \mathbf{x}^T Q\mathbf{x}$$

with Q being certain 6×6 positive definite matrix. Then it is easy to see that Q satisfies the following "Riccati type" equation:

$$P + Q\begin{bmatrix}0 & I\\ A_0 & 0\end{bmatrix} + \begin{bmatrix}0 & I\\ A_0 & 0\end{bmatrix}^T Q - Q\begin{bmatrix}0 & 0\\ 0 & I\end{bmatrix}Q = 0 \tag{16}$$

where $P = w^2 I_{6\times 6}$. A positive definite solution of this "Riccati type" equation exists and is unique. In terms of this solution Q, the solution to the optimal control problem is given by

$$\begin{bmatrix}u_{10}\\ v_0\\ u_{20}\end{bmatrix} = -[0,\, T(\phi)]Q\begin{bmatrix}x_1\\ x_2\end{bmatrix}. \tag{17}$$

From the definition of $T(\phi)$, it is easily seen that while u_{10} and v_0 are nonlinear feedback control, u_{20} is indeed a linear feedback and hence is linearly bounded. This implies that our guess $g_{\max}(\mathbf{x}) = k\|\mathbf{x}\|$ is valid. By Theorem 1, the sufficient conditions that guarantee the above solution is a solution to the robust control problem of (10) are

$$\begin{aligned}2\|v_0\|^2 &\le \beta^2\|\mathbf{x}\|^2\\ \|\epsilon u_{20}\|^2 &\le k^2\|\mathbf{x}\|^2.\end{aligned}$$

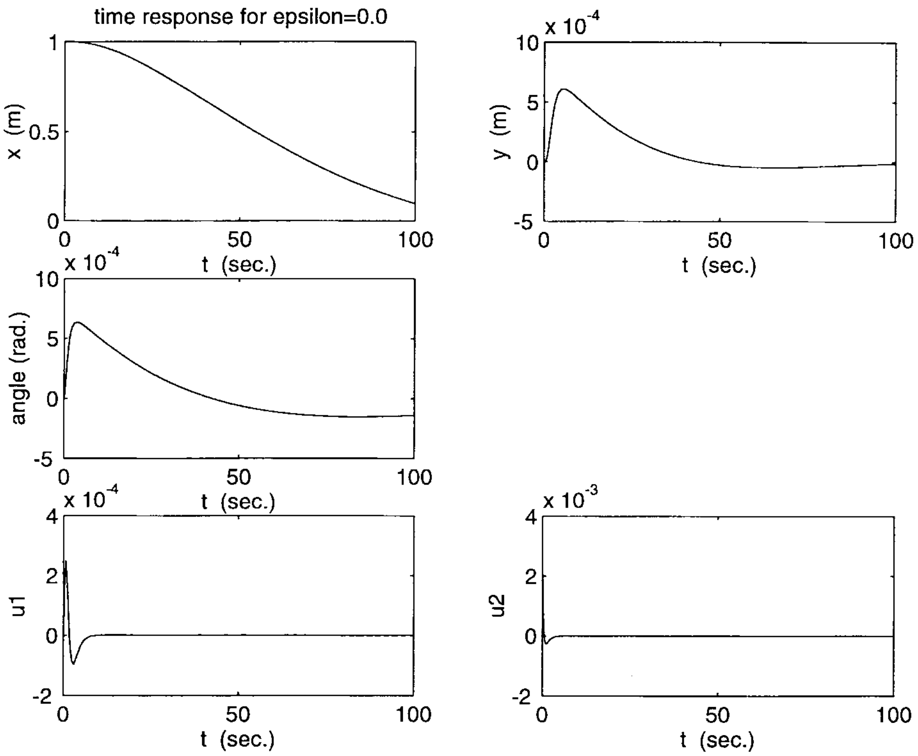

Fig. 2. Response of the translated system for $\epsilon = 0$.

If we take $k = 2, \beta = 2, \phi_0 = 35^o$, and define $w^2 = k^2/\cos^2\phi_0 + \beta^2$, then $w = (k^2/\cos^2\phi_0 + \beta^2)^{1/2} = 10$. The solution to (16) is shown in (17a) at the bottom of the page. Hence

$$\begin{aligned} u_1 =& -\sin\phi(3.1623x - 0.9976\phi + 4.0385\dot{x} - 0.1232\dot{\theta} \\ &+ \cos\phi(3.1623y + 4.0404\dot{y}) \\ u_2 =& -0.0023x + 3.3166\phi - 0.1232\dot{x} + 4.0765\dot{\phi}. \end{aligned} \tag{18}$$

Finally, using (7) and (4) we can obtain the thrust and rolling moment as follows:

$$\begin{aligned} U_t &= mg(1 + u_{10}) - \epsilon_0 J \tan\phi u_{20} \\ U_m &= J u_{20}. \end{aligned}$$

If ϵ_0 is unknown, we will replace it by its nominal value $\bar{\epsilon}_0$:

$$\begin{aligned} U_t &= mg(1 + u_{10}) - \bar{\epsilon}_0 J \tan\phi u_{20} \\ U_m &= J u_{20}. \end{aligned} \tag{19}$$

V. SIMULATION

We will first simulate the following translated system (8), (9) using the control law in (18):

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -\sin\phi \\ \cos\phi - 1 \end{bmatrix} + \begin{bmatrix} -\sin\phi & 0 \\ \cos\phi & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \dfrac{\epsilon}{\cos\phi} \\ 0 \end{bmatrix} u_2$$

$$\ddot{\phi} = u_2.$$

We take the initial conditions

$$\begin{aligned} x(0) &= 1 \\ y(0) &= 0 \\ \phi(0) &= 0 \\ \dot{x}(0) &= 0 \\ \dot{y}(0) &= 0 \\ \dot{\phi}(0) &= 0. \end{aligned}$$

The results for $\epsilon = 0, 0.5$, and 1.0 are shown in Figs. 2–4, respectively (the typical value for ϵ is 0.01). The figures show that the deviation of y and ϕ from zero is very small: $\max\{\phi\} = 0.04°$ and $\max\{y\} = 0.0006$. The control law is quite robust.

$$Q = \begin{bmatrix} 12.7711 & 0 & -0.3990 & 3.1623 & 0 & -0.0023 \\ 0 & 12.7768 & 0 & 0 & 3.1623 & 0 \\ -0.3990 & 0 & 13.5200 & -0.9976 & 0 & 3.3166 \\ 3.1623 & 0 & -0.9976 & 4.0385 & 0 & -0.1232 \\ 0 & 3.1623 & 0 & 0 & 4.0404 & 0 \\ -0.0023 & 0 & 3.3166 & -0.1232 & 0 & 4.0765 \end{bmatrix} \tag{17a}$$

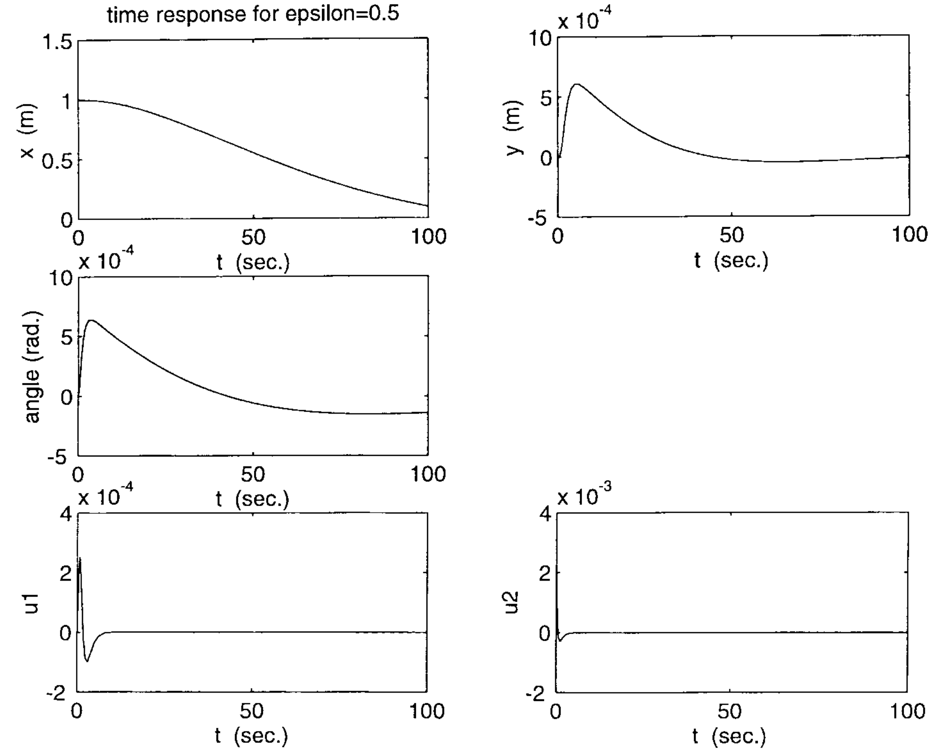

Fig. 3. Response of the translated system for $\epsilon = 0.5$.

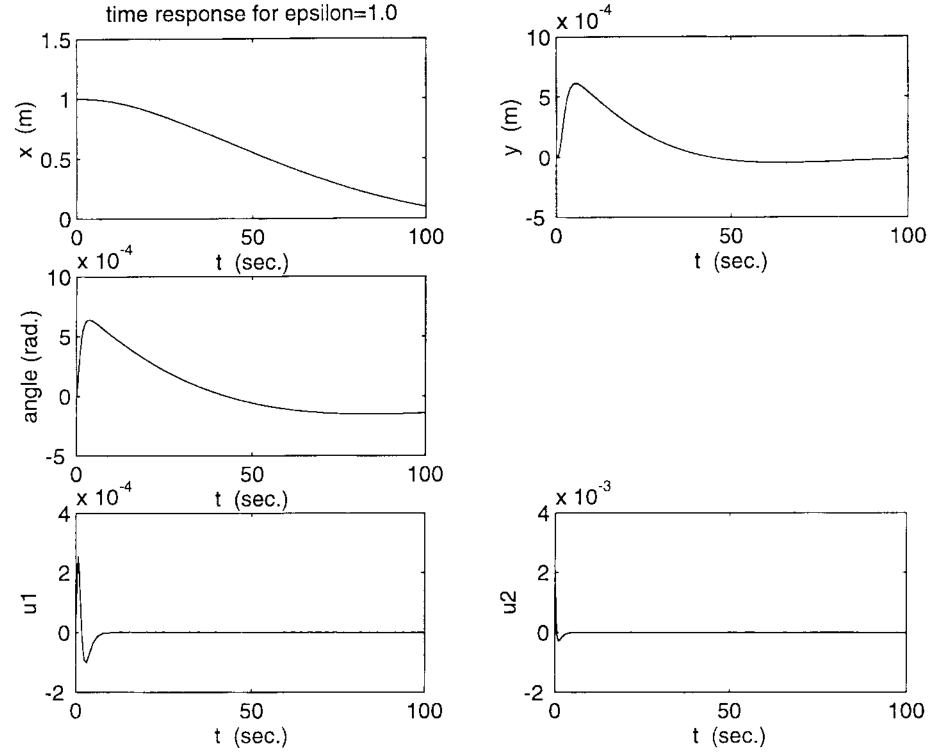

Fig. 4. Response of the translated system for $\epsilon = 1$.

We will also simulate the original system (1)–(3)

$$-\ddot{Y} = \frac{1}{m}(-U_t \sin\phi + \epsilon_0 U_m \cos\phi)$$

$$-\ddot{Z} = \frac{1}{m}(U_t \cos\phi + \epsilon_0 U_m \sin\phi - mg)$$

$$\ddot{\phi} = \frac{1}{J}U_m$$

using the control law given by (18) and (19). We do this because the transformation (7) depends in ϵ and we would like to know the effect of replacing ϵ by its nominal value.

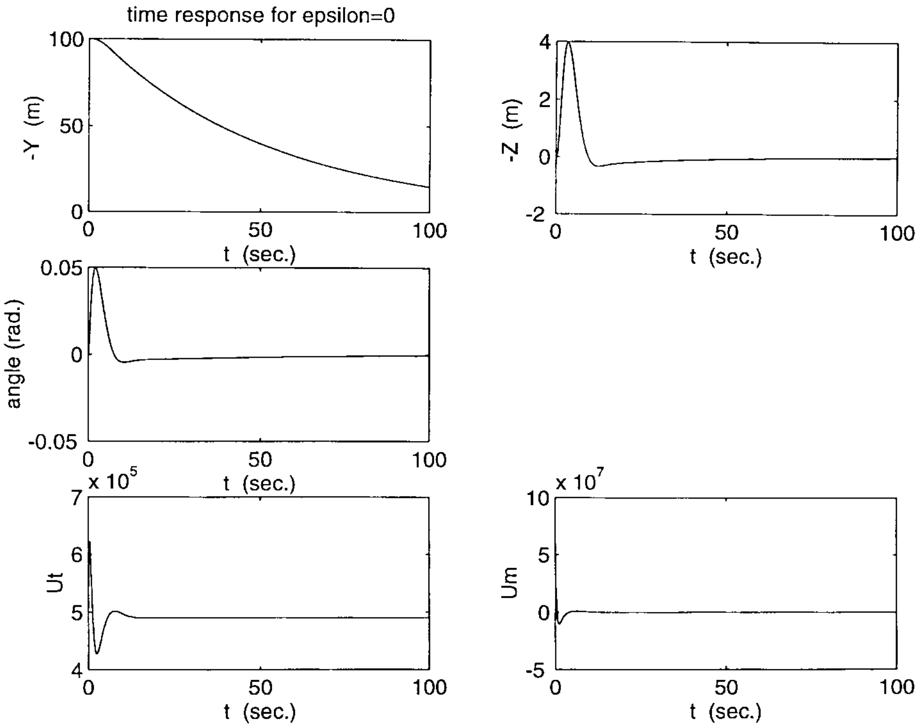

Fig. 5. Response of PVTOL aircraft for $\overline{\epsilon}_0 = 0$.

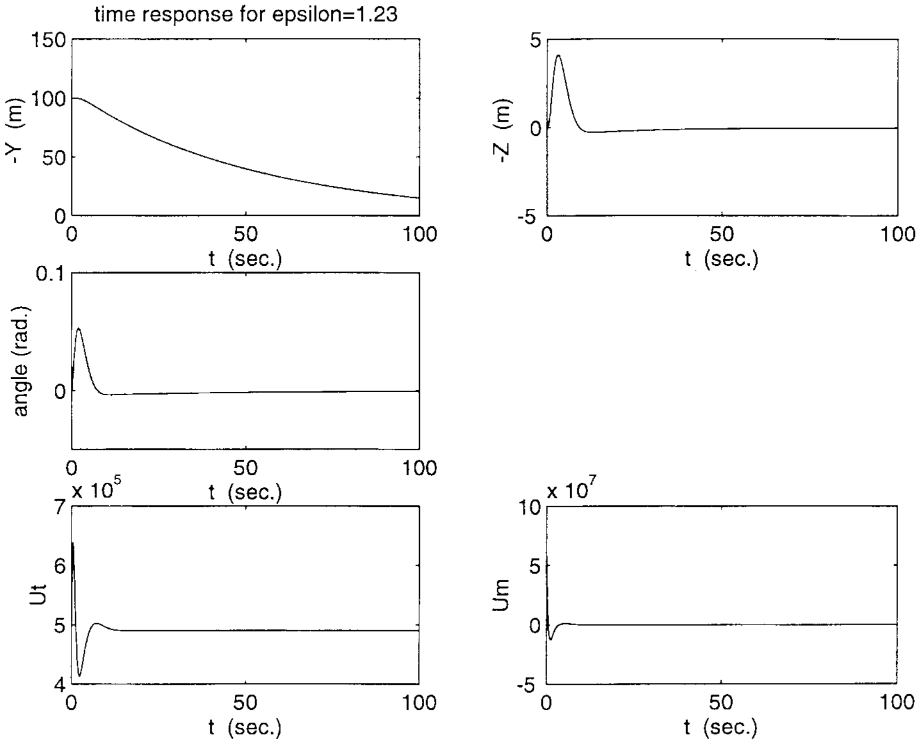

Fig. 6. Response of PVTOL aircraft for $\overline{\epsilon}_0 = 1$.

We use the following values of m and J:

$$m = 5 \times 10^4 \text{ kg}$$

$$J = 2 \times 10^5 \text{ kg} - \text{m}^2.$$

We use the initial condition

$$Y(0) = -100 \text{ m},$$
$$Z(0) = 0 \text{ m}$$
$$\phi(0) = 0^\circ$$

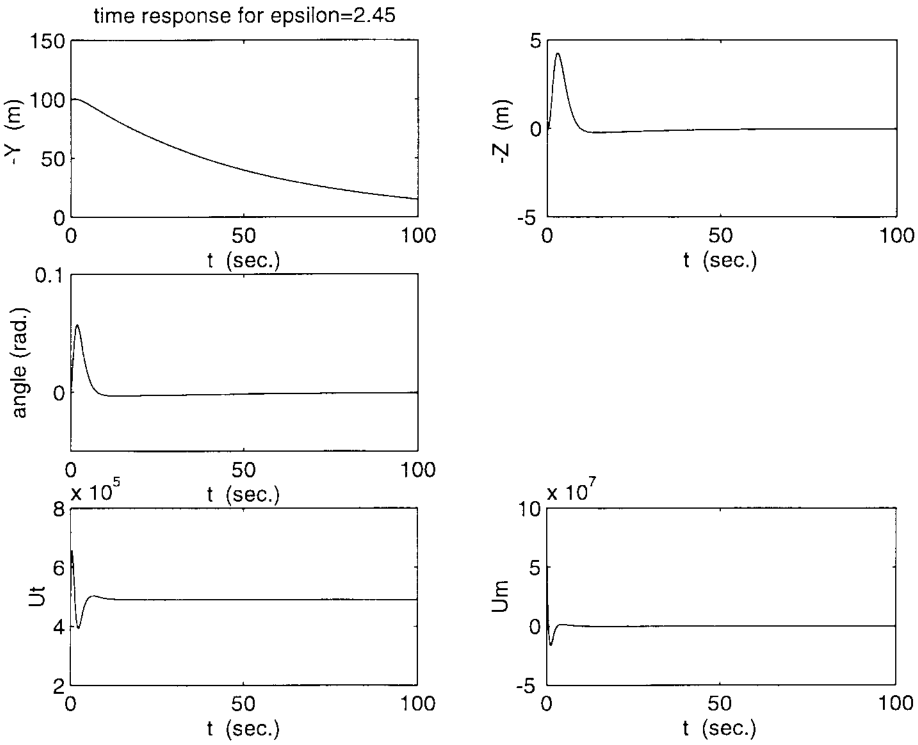

Fig. 7. Response of PVTOL aircraft for $\overline{\epsilon}_0 = 2.5$.

$$\begin{aligned}\dot{Y}(0) &= 0\\ \dot{Z}(0) &= 0\\ \dot{\phi}(0) &= 0.\end{aligned}$$

We take $\overline{\epsilon}_0 = 0.0245$ (corresponding to the typical value $\epsilon = 0.01$) and simulate for $\epsilon_0 = 0.0, 1.0,$ and 2.5. The results are shown in Figs. 5–7, respectively. They indicate that $-Z \geq 0$, $U_t \geq 0$, and $\max\{-Z\} = 5$ m and $\max\{\phi\} = 3°$. Clearly the requirements 1–4 in Section III are indeed satisfied.

VI. DISCUSSION

It should be pointed out that in this work, no effort has been made in optimizing the parameters involved, i.e., choosing the parameter values such that the closed-loop system has the "best" performance. In fact, the choice $\rho = 1$ is by no means the best choice. It is solely for the purpose of being able to solve the Hamilton–Jacobi–Bellman equation (13) explicitly. Intuitively, for better performance, a bigger ρ is preferred, because a smaller weight on v in the cost function results in an optimal control (u_{10}, v_0, u_{20}) which heavily rely on the lateral force v_0, instead of on u_{10} and u_{20}. However, v is the augmented control which is discarded in forming the robust control. Therefore, a more realistic and better robust control law can be obtained by setting $\rho \gg 1$ in the cost function of the corresponding optimal control problem.

The Harrier is a nonminimum phase system. Therefore the theory for explicitly linearizing the input–output response of a nonlinear system using state feedback [1], [3] will not produce a satisfactory control law as indicated in [2]. In fact, one shortcoming of the exact input–output linearization theory is the inability to deal with nonminimum phase nonlinear system.

An approximate input–output linearization procedure, developed for slightly nonminimum phase nonlinear systems was used in [2] to design the hovering control. On the contrary, the method we proposed does not require linearization.

Another approach to aircraft hovering control was proposed in [15]. They using nonlinear regulator theory of [4].

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Feng Lin (S’86–M’87) received the B.Eng. degree in electrical engineering from Shanghai Jiao-Tong University, Shanghai, China, in 1982 and the M.A.Sc. and Ph.D. degrees in electrical engineering from the University of Toronto, Toronto, Ontario, Canada, in 1984 and 1988, respectively.

From 1987 to 1988, he was a Postdoctoral Fellow at Harvard University, Cambridge, MA. Since 1988, he has been with the Department of Electrical and Computer Engineering, Wayne State University, Detroit, MI, where he is currently an Associate Professor. His research interests include discrete-event systems, hybrid systems, robust control, image processing and neural networks.

Dr. Lin is coreceipient of a George Axelby Outstanding Paper Award from the IEEE Control Systems Society. He is also the recipient of a Research Initiation Award from the National Science Foundation, an Outstanding Teaching Award from Wayne State University, a Faculty Research Award from ANR Pipeline Company, and a Research Award from Ford Motor Company. He is an Associate Editor of IEEE TRANSACTIONS ON AUTOMATIC CONTROL.

William Zhang received the B.S. degree in mathematics from Shandong University, Shandong, China. He received the Ph.D. degree in electrical engineering from the University of California at Los Angles in 1990.

He was with GM Tech Center from 1994 to 1997. He is currently a technology architect of IBM Corporation, working for GM, Ford, and Chrysler as clients of consulting. His technical interests include system/product virtual prototyping, embedded controller, and the applications of information technology to engineering, manufacturing, and sales/marketing. His academic and research experience includes working as a Visiting Assistant Professor at the Electrical and Computer Engineering Department of Wayne State University from 1993 to 1994. In the summer of 1993, he worked as a NASA Visiting Research Engineer at Flight Systems Research Laboratory at UCLA. He has published more than two dozen technical papers in refereed journals in the areas of distributed parameter systems, stochastic systems, and nonlinear systems.

Dr. Zhang received the GM VSAS Special Achievement Award for his leadership role in the development of Global Manufacturing Footprint System for Delphi Automotive Systems.

Robert D. Brandt received the B.S. degree in mathematics from the University of Illinois at Chicago in 1982 the M.S. degree in mathematics from the University of Illinois at Urbana-Champaign in 1983 and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of California at Santa Barbara in 1986 and 1987, respectively.

In 1984, he was a resident at the Institut des Hautes Etudes Scientifique in Bures sur Yvette, France. From 1988 to 1990, he was an Assistant Professor of Electrical and Computer Engineering at Wayne State University, Detroit, MI. From 1990 to 1992, He was a Research Scientist at the Environmental Research Institute of Michigan in Ann Arbor, where he was Principal Investigator and Project Manager for a machine-printed character recognition project funded by the United States Postal Service. From 1993 to 1997, he was a Visiting Researcher at the Beckman Institute for Advanced Science and Technology at the University of Illinois at Urbana-Champaign. Currently, he is an Assistant Professor of Computer Systems Engineering at the University of Arkansas. He has published work in the areas of control, machine vision, machine intelligence, neural networks, image processing, and algebraic topology.