Robust and Adaptive Estimation of State of Charge for Lithium-Ion Batteries

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Abstract—Reliable operation of battery management systems depends critically on accurate estimation of the state of charge (SOC) and characterizing parameters of a battery system. SOC estimation employs models that must be robust against variations in battery cell electrochemical features, aging, and operating conditions. This paper reveals that commonly used SOC estimation schemes are fundamentally flawed in providing robustness of SOC estimation against model uncertainties. Parameter estimation methodologies and adaptive SOC estimation design are introduced in this paper to enhance SOC estimation accuracy and robustness. By a scrutiny of the impact of parameter variations on SOC estimation accuracy, the SOC-OCV (open circuit voltage) mapping is identified to be the most critical function that must be accurately established. Identification algorithms are introduced and their convergence properties are established. Integration of the identification algorithms and SOC estimation schemes lead to an adaptive SOC estimation framework that is superior over the existing methods in providing much improved accuracy and robustness. Experimental studies are conducted to validate the algorithms.

Keywords: Lithium-ion battery model; SOC estimation; system identification, robustness; adaption

I. INTRODUCTION

Reliable operation of battery management systems (BMS) \cite{1} plays a pivotal role in safety and power efficiency of electric and hybrid vehicles, and in power management of distributed renewable energy generation and smart grids. The SOC in a battery system is the most important variable to regulate charge/discharge decisions and to ensure the battery’s safety, efficiency, and longevity. As a result, accurate SOC estimation dominates development tasks in BMS design. SOC estimation has been extensively studied by many researchers, battery manufacturers, and automotive companies. Typical methods include inverse mapping using the SOC-to-OCV characterization curves, computation of amp-hours by using load current integration (Coulomb counting), impedance measurements, and Extended Kalman Filters (EKF), H\text{\infty} observers and their modifications \cite{2-11}. To facilitate such estimation schemes, many model structures were introduced \cite{12-17}. SOC estimation employs models that must accommodate variations in battery cell electrochemical features, aging, and operating conditions.

In our recent work \cite{18-20}, identification of model parameters was pursued, using the model structures from \cite{21} and nonlinear models from \cite{22}. This paper uses a different battery model structure \cite{23}, introduces a new SOC-OCV model, and focuses on impact analysis and simplified and reliable algorithms toward practical implementation. A sensitivity analysis on how model parameters affect SOC estimation accuracy concludes that the SOC-OCV mapping is the most critical function that must be accurately established. Identification algorithms are introduced and their convergence properties are established. Integration of the identification algorithms and SOC estimation schemes lead to an adaptive SOC estimation framework that is superior over the existing methods in providing much improved accuracy and robustness.

This paper makes novel contributions in the following aspects to BMS strategies: (i) Sensitivity analysis of model parameters on SOC estimation accuracy. (ii) Demonstration that all observer-based and non-adaptive SOC estimators are non-robust with respect to perturbations on the SOC-OCV mapping. (iii) Demonstration that typical piece-wise linear interpolation models for the SOC-OCV mapping are not identifiable. (iv) Introduction of a modified model structure for the SOC-OCV mapping that is both accurate and identifiable. (v) Identification algorithms and convergence properties for identifying the SOC-OCV mapping. (vi) Introduction of new adaptive SOC estimators. While adaptive observers and joint estimation schemes have been comprehensively explored in our early papers \cite{19, 20} which group all parameters together, the model structure and algorithms of this paper are different and offer reduced complexity by using targeted estimation methods for different parameters to reduce computational complexity and enhance accuracy.

The rest of the paper is organized as follows. Section II describes the battery model structure selected for study in this paper. Section III studies robustness of the observer-based SOC estimators. It reveals that such SOC estimators are fundamentally flawed in providing robustness when model uncertainties are considered. Detailed sensitivity analysis of the
impact of parameter variations on SOC estimation accuracy concludes that the SOC-OCV mapping is the most important function to identify. Model structure improvements are discussed in Section IV. A new model structure is introduced that can accurately represent the mapping and is also identifiable. Identification algorithms are introduced in Section V. Their convergence properties are established and demonstrated. Combining system identification and SOC estimation, Section VI introduces adaptive SOC estimators. The adaptive SOC estimators are validated by using experimental data. Finally, Section VII concludes the paper with a summary of the main findings of this paper and a highlight of related open issues.

II. BATTERY MODELS

Consider the battery model structure in Fig. 1. It is easy to derive its state space representation as

\[
\begin{align*}
\dot{x} &= Ax + Bi \\
v &= C \dot{x} + Ri + a_j, j = 1,..., m
\end{align*}
\]  

(1)

Where \( x = [v_p \ s] \), \( A = \begin{bmatrix} \frac{1}{Q} & 0 \\ 0 & 0 \end{bmatrix} \), \( B = \begin{bmatrix} \frac{1}{Q} \\ 0 \end{bmatrix} \), \( C = [1 \ b_j] \), \( s \) is the state of charge of the battery and \( Q \) is the maximum capacity (Amp-second) \(^1\). We use the positive current \( i \) (A) for charging, and negative values for discharging. Typically, the open-circuit voltage \( v_{OCV} \) (V) is related to \( s \) by a piece-wise linear interpolation function of \( m \) segment where the coefficients \( a_j > 0 \) and \( b_j > 0 \) depend on \( s \). It follows that the terminal voltage \( v \) (V) is

\[ v = v_p + a_j + b_j s + Ri \]  

(2)

![Dynamic battery model](image)

We note that the output equation is an affine function, rather than a linear expression. Also, since \( a_j \) and \( b_j \) are functions of \( s \), the output equation (2) is highly nonlinear. On the other hand, near a given \( s \) this can be transformed into a locally linearized model if the output is changed to a new output variable \( y = v - a_j \), and \( j \) is fixed, resulting in

\[ y = C \dot{x} + Ri \]  

(3)

This paper develops new robust and adaptive SOC estimation schemes that enhance SOC estimation accuracy under parameter uncertainties. For evaluation purposes, a Li-ion battery system of rated capacity 90 Ah is used as a benchmark case for experimental validation. A comprehensive lab testing obtains its nominal parameter values as \( R^0 = 0.0011 \Omega, R_p^0 = 0.0012 \Omega, C_p^0 = 150000 \) F. Consequently, the nominal time constant of the \( R - C \) loop is \( \tau_p^0 = R_p^0 C_p^0 \) = 180 s. From the rated capacity of 90 Ah, the nominal value of the maximum capacity is \( Q^0 = 324000 \) Amp-second.

In our analysis, we use the fact that the two dynamics for \( v_p \) and \( s \) have vastly different time scales. For example, under a constant charge current \( i(t) = i_0 \), the initial condition response for \( v_p(t) \) takes about \( 3 \tau_p^0 \approx 9 \) minutes to diminish \(^2\), and \( v(t) \) reaches its steady state value \( v_p = R_p^0 i_0 \). Consequently, under a relatively smooth charge/discharge current we may use a simplified model to represent the system after this initial transient

\[
\begin{align*}
\dot{s} &= \frac{1}{Q} i_0 \\
v &= a_j + b_j s + (R + R_p) i_0
\end{align*}
\]  

(4)

However, during fast-varying charge current profiles, the dynamics on \( v_p \) cannot be omitted. In our subsequent analysis, we will consider both scenarios.

Due to uncertainties on the system, the true system parameters \( R, R_p, C_p, Q \) are unknown. Consequently, they must be estimated either by offline parameter estimation or real-time system identification, leading to their estimated values \( \hat{R}, \hat{R}_p, \hat{C}_p, \hat{Q} \) that are different from their true values. Denote the estimation errors by

\[
\delta_R = R - \hat{R}, \delta_{R_p} = R_p - \hat{R}_p, \delta_{C_p} = C_p - \hat{C}_p, \delta_{1/Q} = \frac{1}{Q} - \frac{1}{\hat{Q}},
\]

where we use \( \delta_{1/Q} \) rather than \( \delta_Q \), for clarity in error analysis. Similarly, the voltage mapping estimation errors are represented by uncertainties on \( a_j \) and \( b_j \) with \( \delta a_j = \hat{a}_j - a_j \) and \( \delta b_j = \hat{b}_j - b_j \). The main goal of this paper is to develop improved algorithms for SOC estimation under such uncertainties.

\(^1\)The typical unit for \( Q \) is amp-hour (Ah). To be consistent with the time unit “second” in the model, it is modified to amp-second.

\(^2\)It is a commonly accepted guideline that the exponential function \( e^{-\tau t} \) may be considered as near zero after the transient interval of \( 3 \tau \) (actual value 0.0498).
III. CRITICAL ISSUES ON ROBUST SOC ESTIMATION UNDER SMOOTH CHARGE/DISCHARGE CURRENT

Typical usage of battery systems involves both smooth charge/discharge operations in which the current is either a constant or changes its values smoothly and slowly and rapid changes during frequent acceleration and braking. Although the transient behavior introduces certain temporary estimation errors during rapid acceleration and braking, such errors have short duration and limited impact. This will be explained and detailed by using a standard driving cycle in Section VI C. Consequently, we use the simplified model described by the equation (4) in our analysis. There are many ways of designing an estimator for s. The most common structure is the observer

\[
\begin{align*}
\dot{s} &= \frac{1}{q}i_0 - L(\dot{\theta} - v) \\
\dot{\theta} &= \hat{a}_j \hat{b}_j \dot{s} + (\hat{R} + \hat{R}_p) v_0
\end{align*}
\]

where the observer gain L is to be designed. The gain L can be designed by using different methods, such as Kalman filters, pole placement, linear quadratic observers, H∞ observers [24], etc. Each of these observers has its own performance index.

However, we now show, which may seem a shocking surprise, that none of these standard observers can deliver satisfactory performance if the parameter estimation errors are substantial. To understand this, consider the state estimation error e(t) = ̇(t) − s(t) and its dynamics

\[
\begin{align*}
\dot{e} &= \left(\frac{1}{Q} - \frac{1}{Q_0}\right) i_0 - L(\hat{a}_j \hat{b}_j - (a_j + b_j s)) \\
&\quad + ((\hat{R} + \hat{R}_p) - (R + R_p)) v_0 \\
&= -Lb_j e - \delta_{1/Q} i_0 - L(\hat{a}_j \hat{b}_j + \delta_{R} s + \delta_{R_p} i_0)
\end{align*}
\]

which highlights how parameter estimation errors lead to SOC estimation errors.

The estimation error contains two parts: (i) a dynamic transient of \(e^{-\Delta/\Omega} e(0)\) whose time constant 1/(Lb) can be made small (so that this term diminishes fast) if \(L > 0\) is selected to be a big value; and (ii) a steady state error

\[
e_\infty = -\frac{1}{Lb_j} \delta_{1/Q} i_0 - \frac{1}{b_j} (\delta_{a_j} + \delta_{R} \dot{s} + (\delta_{R} + \delta_{R_p}) i_0)
\]

\[\Delta_1 + \Delta_2\]

The first term \(\Delta_1 = -\frac{1}{Lb_j} \delta_{1/Q} i_0\) can be made small by using a large L. In fact this term is usually very small since 1/\(Q_0\) is a very small value. However, the second term

\[
\Delta_2 = -\frac{1}{b_j} (\delta_{a_j} + \delta_{R} \dot{s} + (\delta_{R} + \delta_{R_p}) i_0)
\]

is independent of \(L\).

Example 1 To give a perspective, suppose that the true parameter values are the nominal values \(R^0 = 0.001 \Omega, R_p^0 = 0.0012 \Omega, C_p^0 = 1500000 \text{F}, Q^0 = 324000 \text{Amp-second}\). Also, the nominal values for the output mapping are \(a_j^0 = 3.9\) and \(b_j^0 = 0.25\). The charge is at 0.1C. The operating point of the SOC is \(s = 0.5\). For different \(L\) values, Table I shows the corresponding time constants and steady state errors.

<table>
<thead>
<tr>
<th>(L)</th>
<th>1% Estimation errors</th>
<th>5% Estimation errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time constant 1/Lb_j</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>Steady state[Δ_1]</td>
<td>0.0000011</td>
<td>0.0000011</td>
</tr>
<tr>
<td>Steady state[Δ_2]</td>
<td>0.1668</td>
<td>0.1668</td>
</tr>
<tr>
<td>Time constant 1/Lb_j</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>Steady state[Δ_1]</td>
<td>0.0000055</td>
<td>0.0000055</td>
</tr>
<tr>
<td>Steady state[Δ_2]</td>
<td>0.8340</td>
<td>0.8340</td>
</tr>
</tbody>
</table>

These results show: (1) Since the transient time constants are small, the transient errors can be ignored; (2) \(\Delta_1\) is extremely small and can be omitted in analysis; (3) \(\Delta_2\) is independent of \(L\). In other words, it is futile to seek the observer design to improve SOC estimation under parameter uncertainties. Also, unless the estimation error is very small, SOC estimation accuracy will be completely lost. This is due to the fact that the SOC range is [0, 1] and any error beyond 1 renders a complete loss of SOC estimation accuracy.

Example 1 shows that the only approach for robust estimation of SOC is to estimate parameters more accurately so that \(\Delta_2\) can be reduced. Under the nominal parameter values of Example 1 and 5% parameter estimation errors, these error terms for SOC estimation are

\[
\frac{1}{b_j} \delta_{a_j} = 0.78, \frac{1}{b_j} \delta_{R} \dot{s} = 0.025, \frac{1}{b_j} (\delta_{R} + \delta_{R_p}) i_0 = 0.004.
\]

Apparently, the last term is much smaller than the first two terms.

The main conclusions of the above analysis can be summarized as follows: (i) The standard observer for SOC estimation is incapable of handling model errors. (ii) To deal with model uncertainty, one must estimate the model parameters more accurately. Since battery characteristics vary significantly, change with operating conditions, and deviate with aging, real-time model estimation is inevitable. (iii) The estimation accuracy on \(C_p\) is not critical since it affects only a small transient. As a result, it may be fixed as the nominal value which can be obtained off-line. (iv) The estimation accuracy of \(Q\) is not essential. As a result, it may be fixed as the rated Ah which can be obtained off-line or obtained from the manufacturer. (v) The estimation accuracy of \(a_j\) and \(b_j\) is critical for SOC estimation. This is especially true for estimation of \(a_j\). (vi) The resistance values \(R\) and \(R_p\) should be estimated. However, their impact on SOC estimation accuracy
is not as significant as \(a_i\) and \(b_j\). Also, one may simply estimate \(R + R_p\), rather than individually, since they appear as the summation in \(\Delta t\). (vii) Since estimation of SOC and the parameters must be performed at the same time, methodologies for joint SOC and parameter estimation are mandatory. While this has been pursued in our early work [20], the model structure is different here.

IV. MODEL STRUCTURES FOR PARAMETER ESTIMATION

Although the battery model expressed by the equation (1) involves many parameters and can be identified together in principle [19-26], the task of identifying them can be separated into several groups. This approach is of critical importance in practical implementation of state and parameter estimation for enhanced accuracy and reduced complexity. First, under smooth charge/discharge operations, \(C_p\) does not need to be identified. Furthermore, \(R\) and \(R_p\) can be easily identified. By removing these from the main identification task, the problem is much simplified and estimation accuracy can be better managed with much smaller amount of data.

A. Estimation of \(R\) and \(R_p\)

Estimation of \(R\) and \(R_p\) is quite simple and can be done independently from estimation of SOC or \(a_i\) and \(b_j\). One basic method is the following switching charge approach. Suppose that the constant charge current \(i_0\) has been used for a period of time so that \(v_p(t)\) has reached its equilibrium point. In this case, the voltage relation holds \(v(t) = v_{OCV}(t) + (R + R_p)i(t)\).

At \(t_0\), the charge current is tentatively decreased from \(i_0\) down to 0. As a result, \(v_{OCV}(t) = v_{OCV}(t_0), \ t \geq t_0\). Wait until \(v(t)\) reaches its new steady state which is precisely \(v_{OCV}(t_0)\).

Then \(R_{sum} = R + R_p\) can be estimated as \(R_{sum} = \frac{v_{OCV}(t_0) - v(t_0)}{i_0}\).

This procedure is illustrated in Fig. 2. Since this estimation is very easy to implement, quite accurate, and independent of estimation of other parameters, in the subsequent development, we assume that \(R_{sum}\) is known.

\[ y(t) = v(t) - (R + R_p)i(t) = a_j + b_js(t) \]

B. Deficiency of Interpolation Models in Joint Estimation of SOC and Parameters

We now develop a new algorithms for estimating jointly SOC, \(Q\), \(a_i\) and \(b_j\), assuming that \(R_{sum}\) is known. Suppose that at \(t = 0, s = s_0\) is unknown. From the relationship

\[ s(t) = s_0 + \frac{1}{Q} \int_0^t i(\tau) d\tau \]

the estimation of \(s(t)\) can be reduced to that of \(s_0\) and \(Q\) since \(i(t)\) is measured. Let \(q(t) = \int_0^t i(\tau) d\tau\). Re-organize the equation (4) into

\[
\begin{cases}
    s(t) = s_0 + \frac{1}{Q} q(t) \\
    y(t) = v(t) - (R + R_p)i(t) = a_j + b_j s(t)
\end{cases}
\]

where implies

\[
y(t) = a_j + b_j \left( s_0 + \frac{1}{Q} q(t) \right) = a_j + b_j s_0 + \frac{b_j}{Q} q(t)
\]

Apparently, in this model structure, we can identify \(a_j + b_j s_0\) and \(\frac{b_j}{Q}\) but not SOC and the parameters individually. In a piece-wise interpolation model, each linear segment encounters two parameters. As a result, even if we extend the data range to cover several segments, the model remains unidentifiable. This analysis implies that although the piece-wise linear interpolation model for the SOC-OCV mapping is easy for implementation; it is not suitable for real-time parameter estimation. Consequently, a different and more consolidated structure for the SOC-OCV mapping is needed.

C. Modified Model Structures for SOC-OCV Mapping

For system identification, we need a model structure of the SOC-OCV mapping that has the following desirable properties: (i) It can represent experimental data with high fidelity; (ii) its parameters are identifiable under typical charge/discharge profiles. We should emphasize that in this paper the range of the SOC is \([0, 1]\) which is the “usable” SOC, excluding the ranges of over-charge or over-discharge. The usable range is determined by the protective circuits which limit the charge and discharge to certain predetermined upper and lower thresholds on OCV.

Example 2 The SOC-OCV relationship is plotted as the solid line in Fig. 3. The experimental data show three characteristic sections: (i) For the high SOC values over 0.92, the curve is fast rising; (ii) In the middle range of the SOC in \([0.15, 0.92]\), the curve is nearly linear; (iii) When the SOC drops below 0.15, the curve shows a fast voltage drop. Based on these observations, our model structure consists of three functions (exponential-linear-logarithm with fractional power):

\[
y = v - (R + R_p)i = a - b(-\ln(s))^{2.1} + cs + de^{30(s-1)}
\]

where \(y\) is OCV, \(s\) is the SOC, and \(a, b, c,\) and \(d\) are model parameters to be determined. For the data in Fig.3, a least squares data fitting results in model parameters \(a = 3.81, b = 0.022, c = 0.31, d = 0.07\). The model is illustrated as the point curve in Fig. 3. In the typical operating range of the SOC in \([0.15, 1]\), the fitting error is about 5 mV.
D. Model Identifiability

We now evaluate the critical issue of model identifiability for the equation (9). The third term $e^{30(s-1)}$ in (9) is very small when the SOC is below 0.9. For example, for the battery system in Example 2, $d = 0.07$ and the values are $d e^{30(s-1)} = 0.0035$ if $s = 0.9$, and $d e^{30(s-1)} = 0.0008$ if $s = 0.85$. Since the typical range of the battery system is within $[0.15, 0.9]$, in our system identification algorithm development, we will focus only on the first two terms in (9).

$$y = a - b(-\ln(s))^{2.1} + cs, 0.15 \leq s \leq 0.9 \quad (10)$$

Suppose that the battery system starts with either $s(0) = 0$ or $s(0) = 1$. For concreteness, we assume $s(0) = 0$. In this case, the equation (10) becomes

$$y = a - b\left(-\ln\left(\frac{q(t)}{Q}\right)\right)^{2.1} + c\frac{q(t)}{Q} = f(q(t); \theta) \quad (11)$$

where $q(t) = \int_0^t i(\tau)d\tau$. It contains four unknown parameters $\theta = [a, b, c, Q]$. For $N$ time instances $t_j$, $j = 1, \ldots, N$, we have the data $q_j = q(t_j)$ and $v_j = v(t_j)$, leading to $N$ observation equations

$$\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} f(q_1; \theta) \\ \vdots \\ f(q_N; \theta) \end{bmatrix} = F_N(\theta) \quad (12)$$

The Jacobian matrix for this mapping is

$$J_N = \frac{\partial F_N(\theta)}{\partial \theta} = \begin{bmatrix} 1 - \left(-\ln\left(\frac{q_1}{Q}\right)\right)^{2.1} & \frac{q_1}{Q} & -2.1b \left(-\ln\left(\frac{q_1}{Q}\right)\right) \frac{1}{Q} - c\frac{q_1}{Q^2} \\ \vdots & \vdots & \vdots \\ 1 - \left(-\ln\left(\frac{q_N}{Q}\right)\right)^{2.1} & \frac{q_N}{Q} & -2.1b \left(-\ln\left(\frac{q_N}{Q}\right)\right) \frac{1}{Q} - c\frac{q_N}{Q^2} \end{bmatrix} \quad (13)$$

To ensure identifiability, the Jacobian matrix must be full rank. In fact, for fast convergence, it is desirable that $J_N$ has a good condition number. The condition number of a Jacobian matrix is a common measure of numerical difficulty in solving an inverse problem. Since system identification and state estimation are inherently inverse problems, we use this as a measure to compare different estimation strategies. For problems with large condition numbers, accurate estimation requires large data and more accurate measurement devices, both of which increase complexity and cost. We will use several examples to evaluate $J_N$ under different conditions.

Example 3 Consider the same battery system as in Example 2, with the true values $a = 3.81$, $b = 0.022$, $c = 0.31$, $d = 0.07$, $Q = 324000$. From $s(0) = 0$ and charged at 9 A (0.1C), suppose that the data collection starts at $t_1 = 5400$ second ($s(t_1) = 0.15$), collects every 1000 second, and ends at $t_N = 32400$ second ($s(t_N) = 0.9$). Correspondingly, $N = 28$.

(i) Only $Q$ is unknown: In this case, $J_N$ in the equation (13) is reduced to its last column,

$$J_N^Q = \frac{\partial F_N(\theta)}{\partial \theta} = \begin{bmatrix} -2.1b \left(-\ln\left(\frac{q_1}{Q}\right)\right)^{1.1} \frac{1}{Q} - c\frac{q_1}{Q^2} \\ \vdots \\ -2.1b \left(-\ln\left(\frac{q_N}{Q}\right)\right)^{1.1} \frac{1}{Q} - c\frac{q_N}{Q^2} \end{bmatrix} \quad (14)$$

The condition number of $J_N^Q$ is 1.

(ii) Only $a$ and $b$ are unknown: In this case, $J_N$ in (13) is reduced to its first two columns,

$$J_N^{ab} = \frac{\partial F_N(\theta)}{\partial \theta} = \begin{bmatrix} 1 & -\left(-\ln\left(\frac{q_1}{Q}\right)\right)^{2.1} \\ \vdots & \vdots \\ 1 & -\left(-\ln\left(\frac{q_N}{Q}\right)\right)^{2.1} \end{bmatrix} \quad (15)$$

The condition number of $J_N^{ab}$ is 2.2589.

(iii) Only $c$ is unknown: In this case, $J_N$ in (13) is reduced to its third column,

$$J_N^c = \frac{\partial F_N(\theta)}{\partial \theta} = \begin{bmatrix} \frac{q_1}{Q} \\ \vdots \\ \frac{q_N}{Q} \end{bmatrix} \quad (16)$$

The condition number of $J_N^c$ is 1.

(iv) $a$, $b$, $c$ are unknown, but $Q$ is known: In this case, $J_N$ in (13) is reduced to its first three columns,

$$J_N^{abc} = \frac{\partial F_N(\theta)}{\partial \theta} = \begin{bmatrix} 1 & -\left(-\ln\left(\frac{q_1}{Q}\right)\right)^{2.1} & \frac{q_1}{Q} \\ \vdots & \vdots & \vdots \\ 1 & -\left(-\ln\left(\frac{q_N}{Q}\right)\right)^{2.1} & \frac{q_N}{Q} \end{bmatrix} \quad (17)$$

The condition number of $J_N^{abc}$ is 17.4283.

(v) All four parameters are unknown: In this case, the full-scale $J_N$ in (13) is in effect. The Jacobian matrix is full rank. Further evaluation reveals that the condition number of $J_N$
is $7.66 \times 10^8$. This is a high value, but remains a numerically manageable condition. However, for practical implementation, it is desirable to find ways to identify parameters partially so that online estimation complexity can be reduced.

In the above examples, we conclude that in all scenarios, the parameters are identifiable. In the last case when all parameters are unknown and must be identified together, the condition number is high, indicating that more careful numerical solutions need to be sought to ensure identification accuracy. In contrast, in all other cases when some parameters are known, the condition numbers are small, and hence parameter convergence is relatively easy to obtain. These will be verified in subsequent identification algorithms.

V. IDENTIFICATION ALGORITHM

A. Capacity Estimation

We start with the case of capacity estimation. Assume that the only unknown parameter is $Q$. Consider the data set in equation (12), with data points $q_j = q(t_j)$ and $y_j = y(t_j)$, $j = 1, ..., N$. Suppose that the estimate of $Q$ at the $n$th iteration step is $Q_n$. Denote $y = [y_1 \ldots y_N]'$ and

$$y_{est}(Q_n) = \left[ a - b \left( - \ln \frac{q_1}{Q_n} \right)^{2.1} - c \frac{q_1}{Q_n} \right]$$

$$\vdots$$

$$y_{est}(Q_n) = \left[ a - b \left( - \ln \frac{q_N}{Q_n} \right)^{2.1} - c \frac{q_N}{Q_n} \right]$$

$$f_N^{Q_n} = \left[ -2.1b \left( - \ln \frac{q_1}{Q_n} \right)^{1.1} \frac{1}{q_n} - c \frac{q_1}{q_n^2} \right]$$

$$\vdots$$

$$f_N^{Q_n} = \left[ -2.1b \left( - \ln \frac{q_N}{Q_n} \right)^{1.1} \frac{1}{q_n} - c \frac{q_N}{q_n^2} \right]$$

(18)

Starting from an initial estimation $Q_0$, $Q_n$ is iteratively updated by the algorithm

$$Q_{n+1} = Q_n - \mu \left( f_N^{Q_n} \right)^{-1} f_N^{Q_n} y_{est}(Q_n) - y$$

(19)

where $\mu$ is the step size. This is a gradient-based iterative search algorithm, with a tunable step size, $\mu$ needs to be selected on the basis of the specific battery model.

Example 4 Consider the same battery system as in Example 3. From $s(0) = 0$ and charged at 9 A (0.1C), suppose that the data collection starts at $t_1 = 5400$ second ($s(t_1) = 0.15$), collects every 2000 second, and ends at $t_N = 31400$ second ($s(t_n) = 0.8722$). Correspondingly, $N= 14$. The observation data are listed in Table II.

<table>
<thead>
<tr>
<th>$q$ (As)</th>
<th>$y$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48600</td>
<td>3.772</td>
</tr>
<tr>
<td>66600</td>
<td>3.816</td>
</tr>
<tr>
<td>84600</td>
<td>3.850</td>
</tr>
<tr>
<td>102600</td>
<td>3.878</td>
</tr>
<tr>
<td>120600</td>
<td>3.903</td>
</tr>
</tbody>
</table>

With the step size $\mu = 0.01$ and the initial estimate $Q_0 = 80$ Ah which is an under-estimate of the true value 90 Ah, the algorithm (19) is executed 50 steps. The estimates and the true capacity values are shown in the left plot of Fig. 4. The right plot shows the estimates when the initial estimate is an over-estimate of 100 Ah. In both cases, the algorithm shows fast convergence.

B. Output Mapping Estimation

Suppose that the capacity $Q$ is known, but the output mapping must be estimated. This amounts to estimation of $a$, $b$, and $c$. Consider the data set in (12), with data points $q_j = q(t_j)$ and $y_j = y(t_j)$, $j = 1, ..., N$. Suppose that the true parameter vector is $\theta = [a \quad b \quad c]'$, and the estimate of $\theta$ at the $n$th iteration step is $\theta_n$. Denote $y = [y_1 \ldots y_N]'$ and

$$f_N^{abc} = \left[ 1 - \left( - \ln \frac{q_1}{Q} \right)^{2.1} \frac{q_1}{Q} \right]$$

$$\vdots$$

$$f_N^{abc} = \left[ 1 - \left( - \ln \frac{q_N}{Q} \right)^{2.1} \frac{q_N}{Q} \right]$$

$$y_{est}(\theta_n) = f_N^{abc} \theta_n$$

(20)

It is noted that this is actually a linear estimation problem. As a result, $f_N^{abc}$ does not depend on $\theta_n$.

Starting from an initial estimation $Q_0$. $Q_n$ is iteratively updated by the algorithm

$$\theta_{n+1} = \theta_n - \mu \left( f_N^{abc} \right)^{-1} f_N^{abc} y_{est}(\theta_n)$$

(21)
Example 5 Consider the same battery system and data set as in Example 3. Suppose that \( Q \) is known, but \( a, b, \) and \( c \) are unknown and must be estimated. We will denote the true parameter set as \( \theta = [a \ b \ c]^T = [3.81 \ 0.022 \ 0.31]^T \). The estimate of \( \theta \) at the \( n \)th iteration step is denoted by \( \hat{\theta}_n \).

The initial estimate is \( \hat{\theta}_0 = [3.6 \ 0.03 \ 0.4]^T \) with an initial estimation error \( \| \hat{\theta}_0 - \theta \|_2 = 0.2286 \). The estimation algorithm expressed by equation (21) is applied with step size \( \mu = 0.01 \). The estimation error trajectory is shown in Fig. 5.

VI. ADAPTIVE SOC ESTIMATION

Typical SOC estimation methods include coulomb counting and observers. It is known that the coulomb counting method can be used if one can accurately estimate initial SOC and capacity. Without real-time battery estimation, this method cannot be used alone.

A. Non-adaptive Observers

The observer-based design is currently a dominant approach in BMS design [25-27]. In particular, the Kalman filter and its extended versions are widely employed. Thanks to their feedback structures, the observer based design has inherent robustness against parameter uncertainties on the state equations, if the output equation is accurate. On the other hand, as revealed by our previous analysis, the observer-based design fails when the output equation contains uncertainties.

In the following demonstration, to focus on the main issues we use a full-order observer. The observer is

\[
\begin{align*}
\dot{\hat{x}} &= \hat{A}\hat{x} + \hat{B}i - L(\hat{\theta}(t) - \nu(t)) \\
\hat{\nu} &= \hat{a} - \hat{b}(-\ln(\hat{x}_2))^{2.1} + \hat{c}\hat{x}_2 + \hat{d}e^{30(\hat{x}_2-1)} + \hat{R}i + \hat{x}_1
\end{align*}
\]

(22)

where all system parameters are replaced by their estimates. Here, the observer gain vector is \( L \) that can be designed by many different methods, such as pole placement, Kalman filters, H\( \infty \) observers, among others. Since the actual design is not critical here, we will choose a fixed \( L \) that demonstrates satisfactory SOC estimation under accurate models.

Example 6 Consider the same battery system and data set as in Example 3. The initial SOC is \( x_2(0) = 0.3 \) and \( v_p(0) = 0 \). The observer feedback gain is chosen as \( L = [0.1 \ 1]^T \). The observer starts with a wrong SOC value \( \hat{x}_2(0) = 0.4 \). Under the accurate battery model, Fig. 6 shows that after a relatively short transient, SOC estimation becomes accurate.

To understand the impact of parameter uncertainties on the SOC estimation accuracy, we now consider variations of parameters. We emphasize that the output equation is affected by the parameters \( a, b, c, d, R, R_p \). In the range of this evaluation, \( d \) has no effect, and hence will not be considered. On the other hand, \( Q \) is an internal parameter that is part of the state equation. We consider perturbations of these parameters and evaluate their impact on SOC estimation. Fig. 7 illustrates such impact by each parameter perturbation individually while the others are assumed to be accurate. The perturbed parameters are \( \hat{a} = 3.85, \hat{b} = 0.024, \hat{c} = 0.341 \) (from \( c = 0.31 \)), \( \hat{Q} = 0.35, \hat{R} = 0.0011, \hat{R}_p = 0.00132 \).

From this example, we may reach the following conclusions, which are consistent with our previous analysis. (i) If the output equation is accurate, then accuracy of \( Q \) is not important for SOC estimation, see plot (a) in Fig. 7. The reason is that the output equation will provide accurate information to correct SOC estimation errors even when the capacity is estimated wrong. (ii) Although \( R \) and \( R_p \) affect SOC estimation accuracy, their impact is negligible, see plots (e) and (f) in Fig. 7. This is due to their very small values. Under the 0.1 C charge current of 9 A, the combined voltage drop on both resistors is only 0.0198 V. (iii) The SOC-OCV curve is critically important, see plots (b), (c), (d) in Fig. 7. Perturbation to these functions causes significant loss of SOC accuracy. Consequently, major effort in improving SOC estimation accuracy must be placed on these functions.

Fig.5 Estimation errors of the SOC-OCV mapping

Fig.6 SOC estimation by the observer design

Fig.7 SOC estimation by the observer design: Perturbation analysis
B. Adaptive SOC Estimation

To ensure SOC estimation accuracy, the critical task is to estimate \(a, b,\) and \(c\) more accurately. This has been discussed in the part B of Section V with the estimation algorithm (21). From a practical viewpoint, the SOC-OCV curve of a battery system changes slowly over time when battery electrochemistry properties vary due to aging or other factors. Consequently, estimation of \(a, b,\) and \(c\) may be done as a periodic updating task so that the SOC observer can be updated with new estimates of \(a, b,\) and \(c\). Estimation of \(a, b,\) and \(c\) can be accomplished by first fully charging it so that the initial SOC becomes \(\hat{s}(0) = 1\). Since estimation of \(\hat{A}\) and \(\hat{B}\) are not essential in SOC estimation accuracy, in our adaptive SOC estimator, they will be kept as their nominal values \(A\) and \(B\). Also, the estimates for \(R\) and \(R_p\) have very limited impact of SOC estimation accuracy. They will also be kept at their nominal values as \(\overline{R}\), etc.

\[
\begin{align*}
(1) & \text{Estimation of } \theta = [a \ b \ c]' \\
& \theta_{n+1} = \theta_n - \mu ((J_H^{abc}) J_B^{abc})^{-1} (J_H^{abc} \theta_n - y) \quad (2) \text{SOC Estimator Adaptation and Implementation}
\end{align*}
\]

\[
\hat{x} = \hat{A}\hat{x} + \hat{B}i - L(\hat{v}(t) - v(t))
\]

\[
\hat{v} = \hat{a} - \hat{b}(-\ln(\hat{x}_2))^{2.1} + \hat{c}\hat{x}_2 + \hat{d}e^{30(\hat{x}_2-1)} + \hat{R}i + \hat{x}_3
\]

(23)

This algorithm contains two parts. The first one is a gradient search algorithm for parameter updating. Then, the second part uses the updated parameters to adapt the SOC-OCV mapping in estimation of SOC. The structure is a recursive framework of the adaptive observer. The flowchart of the proposed estimation algorithm is illustrated in Fig.8.

C. Experimental Validations

We now use experimental data to validate the adaptive SOC estimator represented by the equation (23). The battery testing platform is shown in Fig. 9. The instrumentations used to perform the experiments in this paper are the Arbin Testing System, thermostat, and PC. An Arbin Instrument BT2000 battery testing system was used to carry out the charge and discharge test. The maximum voltage and charge/discharge current of the equipment are 5V and 400A, respectively, in which the current can be set in low range (-1A~1A), middle range (-50A~50A), or high range (-400A~400A), according to the required maximum current. It can be operated in constant current mode, constant voltage mode, constant power mode and dynamic charging and discharging mode. The volume of the thermostat is about 0.5m³. The measuring temperature range is 233.15K-373.15K. The battery used in this study is LiMn₂O₄ cathode and graphite anode lithium ion batteries. Its nominal capacity is 90Ah with a weight of 2.8 kg. The dimension of the battery is 346 mm × 255 mm × 18 mm. The battery was placed in the thermostat to keep the battery at desired temperature.

<table>
<thead>
<tr>
<th>Table III: Data points</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
</tr>
<tr>
<td>True values</td>
</tr>
<tr>
<td>(\theta_{100})</td>
</tr>
<tr>
<td>(\theta_{300})</td>
</tr>
<tr>
<td>(\theta_{500})</td>
</tr>
</tbody>
</table>

Example 7 Here we use the estimation results of Example 5 to show how SOC estimation accuracy improves when the estimates of \(a, b,\) and \(c\) become more accurate. The estimated values of \(a, b,\) and \(c\) are listed in Table III, corresponding to the estimates obtained at the iteration steps 100, 300, 500, respectively.

![Fig.9 The battery testing platform](image-url)
Fig. 10 Adaptive SOC estimation accuracy vs. estimates of $\theta$ under the DST profile

The Dynamic Stress Test (DST) which was introduced in the USABC manual was used to validate the proposed adaptive SOC estimation method. From Table III, it can be seen that the estimated values of parameters $a$, $b$, and $c$ converge to the true values after 300 iteration steps. Fig. 10 demonstrates that when the estimates of parameters $a$, $b$, and $c$ become more accurate, SOC estimation becomes more reliable at dynamic charge/discharge profiles. Fig. 11 shows that the SOC steady-state estimation errors can be controlled approximately 3% using the estimated parameters. The accuracy can be further improved when the model parameters such as $r$, $R_g$ change as SOC varies. The estimated SOC converges fast to the true value; the estimation error falls within 5% after 200 seconds starting from the initial SOC error of 50%. Fig. 12 shows the terminal voltage estimation error with the estimated parameter values after 300 iterations. Combined with Fig. 11, it is found that the proposed adaptive algorithm can get high accurate SOC estimation even though the simulation terminal voltage error is enlarged. It is concluded that the proposed adaptive SOC estimation method can significantly improve the estimation accuracy and convergence speed, which also can be easily implemented in battery management systems.

The adaptive SOC estimator introduced in this paper is one effective remedy to this issue and a base for developing more reliable BMS control functions. The methodologies of this paper can be extended to different battery types and model structures. Other factors of uncertainties in battery systems such as measurement noises, thermal models and battery degradations are also important for reliable BMS development and will be reported in separate papers.

**REFERENCES**


