Mesh Analysis Examples

Use mesh analysis to find the mesh currents $I_1$ and $I_2$.

Note: What is the voltage across the current source? This is an unknown. In order to sum voltages around mesh 1 and mesh 2, we will assume a voltage across the 2A current source. Call this voltage $V_s$.

Constraint equation: Since the current source is in a branch that carries two mesh currents, it must be true that

$$2 = I_s - I_1 \quad [2 \text{ in direction of } I_2]$$
The system of equations:

Constraint equation:

\[-I_1 + I_2 = 2\]

Mesh 1:

\[-10 + 10I_1 + V_s + 20I_1 = 0\]

Mesh 2:

\[50I_2 - 20 + 10I_2 - V_s = 0\]

Simplify:

Mesh 1:

\[30I_1 + V_s = 10\]

Mesh 2:

\[60I_2 - V_s = 20\]

3 equations, 3 unknowns:

\[
\begin{bmatrix}
30 & 0 & 1 \\
0 & 60 & -1 \\
-1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
V_s
\end{bmatrix} =
\begin{bmatrix}
10 \\
20 \\
2
\end{bmatrix}
\]

\[I_2 = 1\ A\]

\[V_s = 40\ V\]
**Mesh Analysis Examples**

**Mesh 1**

\[-10 + 10I_1 + V_3 + 20I_2 = 0\]

**Mesh 2**

\[50I_2 - 20 + 10I_2 - V_5 = 0\]

Add these equations to eliminate \(V_5\).

-10 + 10I_1 + 50I_2 - 20 + 10I_2 + 20I_1 = 0

Look at the circuit. Trace the chain of voltage drops described by the equation.

This is a loop that excludes the branch with the current source.
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Simplify the resultant loop equation:

\[30I_1 + 60I_2 = 30\]

This and the constant equation:

2 equations, 2 unknowns.

\[
\begin{bmatrix}
30 & 60 \\
-1 & 1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
30 \\
2
\end{bmatrix}
\]

\[I_1 = -1\ A\]

\[I_2 = 1\ A\]