Error Analysis of Digital Phase Measurement of Distorted Waves

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Abstract—This paper presents the error analysis of phase measurement of distorted waves using existing phase meters. In one of the most recently published papers on the phase meter [1], phase is measured by first converting the two input waves into two square waves and then measuring the time difference between the pulse centers of these two square waves. This technique is better than the conventional technique, where the phase is computed by measuring the time difference between the zero crossing points [4], [5]. However, this technique may introduce large errors for some particular types of input waves. The main purpose of this paper is to explain how large the error could be under certain conditions on the input waves. The phase meter of [1] can be used when the phase can take values between 0 and 180 deg. Another paper [2] presents a technique for phase measurement where the phase can take values between −180 and 180 deg.

I. INTRODUCTION

A number of phase measurement techniques are presented in [1]–[6]. In [1] the phase between two waves of the same frequency is computed by first converting the waves into two square waves and then measuring the time difference between the pulse centers of these square waves. This technique is better than the conventional techniques explained in [4]–[6]. However, this improved technique may produce a very large phase error when an input wave is distorted by a harmonic. Ibrahim and Abdul-Karim [1] have explained five different error sources in their paper. They claimed that the errors due to some of the sources are negligible, and are within 0.5 percent due to other sources. However, the effect of distortion on the phase error was not explained, i.e., how large the error could be due to the distortion of the fundamental by a harmonic or a number of harmonics. Thus readers may think that the meter is also equally reliable for distorted waves. Moreover, the technique presented in [1] for measuring the time difference between the pulse centers of two square waves is not correct. Wagdy and Lucas [3] mention that their phase measurement technique can be used for distorted waves when the waves are distorted by even harmonics, and that an even out-of-phase harmonic produces a waveform which is symmetrical about its peak value. Thus their corrective action still operated correctly. Actually, this is not true as an even out-of-phase harmonic does produce some error in phase measurement.

The main purpose of this paper is to illustrate the effect of distortion on the phase error. It presents computer simulation results for a number of different cases to show how large the phase error could be due to the distortion of a wave by a harmonic. The simulation software was run on a VAX 11/780 computer. Users of these types of phase meters will be able to benefit from the simulation results. By looking at the simulation results one can easily decide whether the error present in a given application of the meter would be acceptable.

II. PERFORMANCE OF ONE EXISTING TECHNIQUE FOR PHASE MEASUREMENT

The principle of operation of the existing meter presented in [1] is shown in Fig. 1(a). In all the figures the symbols $A_i$ and $\Phi_i$ are used to represent the amplitude and
Fig. 2. Simulation results for the technique presented in [1]. Here \( v_2 \) is distorted by a second harmonic. (a) Phase error versus fundamental phase of \( v_2 \). (b) Maximum phase error versus second harmonic phase of \( v_2 \).

The phase of the \( i \)th harmonic in wave \( v_2 \), respectively. The equations of the waves \( v_1 \) and \( v_2 \), shown in Fig. 1, are

\[
v_1 = A \sin (\omega t)
\]

\[
v_2 = A_1 \sin (\omega t + \Phi_1) + A_2 \sin (2\omega t + \Phi_2)
\]

where \( A_2/A_1 = 0.2 \) for Fig. 1(a) \( \Phi_1 = -60^\circ \) and \( \Phi_2 = 40^\circ \), for Fig. 1(b) \( \Phi_1 = 0^\circ \) and \( \Phi_2 = 40^\circ \), and for Fig. 1(c) \( \Phi_1 = 180^\circ \) and \( \Phi_2 = 40^\circ \). The square waves \( m_1 \) and \( m_2 \) are generated from the input waves \( v_1 \) and \( v_2 \).

The phase difference between \( v_1 \) and the fundamental component of \( v_2 \) is computed by measuring the time difference between the pulse centers of \( m_1 \) and \( m_2 \). This will give an accurate phase value when the peak of the fundamental component of \( v_2 \) appears at the pulse center of \( m_2 \); otherwise, the measurement will introduce some error in the phase angle. Square wave \( m_3 \) is generated by doing an exclusive-OR operation between \( m_1 \) and \( m_2 \). Wave \( m_3 \) has two pulses of widths \( t_r \) and \( t_e \) within every cycle of the fundamental, where \( t_r \) and \( t_e \) are known as the positive and negative zero cross time difference, respectively. From Fig. 1(a) it is clear that

\[
t_r = t_T + a - b
\]

\[
t_e = t_T + b - a
\]

where \( t_T \) is the time difference between the pulse centers of waves \( m_1 \) and \( m_2 \). From (2a) and (2b) the value of \( t_T \) can be expressed as

\[
t_T = (t_r + t_e)/2.
\]

In [1], the above equation was used to determine the time difference between the pulse centers of waves \( m_1 \) and \( m_2 \). The phase angle \( \Phi \) between \( v_1 \) and the fundamental of \( v_2 \) can be expressed, in degree, as

\[
\Phi = 360^\circ \times t_T/T
\]

where \( T \) is the period of the fundamental. In reality, (3) cannot be always used. The reason is explained in Section III.

Figs. 2 and 3 show the computer simulation results of the phase meter presented in [1]. Fig. 2(a) shows the simulation result for the case where \( v_2 \) is distorted by a second harmonic with \( \Phi_2 = 90^\circ \) and \( A_2/A_1 = 0.2 \). This figure shows that the error becomes very large when the phase is close to \( 0^\circ \) or \( 180^\circ \). Similar results were found for the cases where \( v_2 \) is distorted by any other even harmonic. Fig. 2(b) shows the effect of the amplitude of the second harmonic of \( v_2 \) on the maximum phase error. Since for the technique of [1] error becomes maximum when \( \Phi_1 = 0^\circ \) or \( 180^\circ \), the maximum phase errors were computed with \( \Phi_1 = 0^\circ \). Fig. 3 shows simulation results for the case where \( v_2 \) is distorted by a third harmonic. This figure
shows that a third harmonic distortion introduces higher error in phase measurement not only for the cases where the phase is close to 0° and 180°, but also at other values of the phase angle. Similar result was found for other odd harmonic distortion.

III. Description of the Modified Technique for Phase Measurement

In practice (3) will not always measure the correct value of \( t_r \). For example, (3) will not find the correct value of \( t_r \) for the waves \( v1 \) and \( v2 \) shown in Fig. 1(b), and it is clear that

\[
t_f = b - a - t_T
\]

and

\[
t_r = b - a + t_r.
\]

From (5a) and (5b) the value of \( t_r \) can be expressed as

\[
t_r = (t_r - t_T)/2.
\]

Equation (6) is not the same as (3). Thus (3) cannot be used in practice. Equations (3) and (6) could be generalized into a single equation as

\[
t_r = (S_f \cdot t_f + S_r \cdot t_r)/2
\]

where \( S_f \) and \( S_r \) are the signs of \( t_f \) and \( t_r \), respectively. Equation (7) is similar to [3, eq. (3)], but the derivation of (7) is shown here to point out the mistake that was made in [1]. The values of \( S_f \) and \( S_r \) could be either 1 or -1, depending upon the waves \( m1 \) and \( m2 \). The values of \( S_f \) and \( S_r \) could be determined as follows:

*if the rising edge of \( m1 \) is lagging the rising edge of \( m2 \)
  then \( S_f = 1 \), otherwise \( S_f = -1 \)
and if the falling edge of \( m1 \) is lagging the falling edge of \( m2 \)
  then \( S_r = 1 \), otherwise \( S_r = -1 \).

This sign convention will make the value of \( t_r \) positive when the fundamental component of \( v2 \) is leading \( v1 \); otherwise, \( t_r \) will be negative. For example, for the waves of Fig. 1(a) and 1(b), the value of \( t_r \) will be negative, and it can be expressed as

\[
t_r = -(t_f + t_r)/2, \quad \text{for Fig. 1(a)} \]

\[
t_r = -(t_r - t_f)/2, \quad \text{for Fig. 1(b).} \]

Since the value of \( t_r \) could be either positive or negative, this technique could be used to measure the phase when the phase can take values between -180° and 180°, rather than between 0 and 180 deg.

This modified technique does not improve the performance when the distortion is due to an odd harmonic. But it reduces the phase error when the distortion is due to an even harmonic in \( v2 \) and the phase is close to 0°. However, it does not reduce the error when the phase is close to 180°. When the phase is close to 180°, the error could be reduced by further modifying the technique. Fig. 1(c) shows another technique of phase measurement. Here \( t_f \) and \( t_r \) do not represent the positive and negative zero cross time difference, respectively. Thus (7) cannot be used for phase measurement when the phase is close to 180°. A wave \( m4 \) can be generated by doing an equivalence operation between waves \( m1 \) and \( m2 \). Wave \( m4 \) has two pulses of widths \( t_f' \) and \( t_r' \) in every cycle of \( v2 \). The time difference between the pulse centers of \( m1 \) and \( m2' \) (complement of \( m2 \)) can be computed as

\[
t_r = (S_f' \cdot t_f' + S_r' \cdot t_r')/2
\]

where \( S_f' \) and \( S_r' \) are the signs of \( t_f' \) and \( t_r' \), respectively, and these signs are computed as:

- if the rising edge of \( m1 \) is lagging the falling edge of \( m2 \)
  then \( S_f = 1 \), otherwise \( S_f = -1 \)
and if the falling edge of \( m1 \) is lagging the rising edge of \( m2 \)
  then \( S_r = 1 \), otherwise \( S_r = -1 \).

The phase \( \Phi \) between \( v1 \) and the fundamental component of \( v2 \) could be computed as follows:

\[
\Phi = s \cdot (|\Phi_r| - 180°)
\]

where \( s \) equals 1 or -1, depending upon whether \( \Phi \), positive or negative, respectively, and \( \Phi_r \), known as conjugate phase, is computed as

\[
\Phi_r = (360° \cdot t_r')/T.
\]

Simulation results show that if the phase is computed using \( t_r' \), then the error is reduced when the distortion is due to even harmonics and the phase is close to 180° but it does not reduce the error when the phase is close to 0°. Thus the phase measurement algorithm can be written as follows:

**Phase Measurement Algorithm**

1) Compute \( t_f \) using (7);
2) Compute \( t_f' \) using (9);
3) If \( t_r' \leq t_f' \)
   - then compute \( \Phi \) using (4);
   - else compute \( \Phi \) using (10).

IV. Computer Simulation of the Modified Technique

The performance of this modified technique is found to be the same as the performance of the technique presented in [3]. The simulation results are shown in Figs. 4 and 5. Fig. 4(a) shows the simulation result for the case where \( v2 \) is distorted by a second harmonic with \( \Phi_2 = 90° \) and \( A_2/A_1 = 0.2 \). Comparing Fig. 4(a) with Fig. 2(a) it can be concluded that the modified technique has greatly reduced the phase error near 0° and 180° phase angle of the fundamental. Similar results were found for the cases when \( v2 \) is distorted by any other even harmonic. Fig. 4(b) shows the effect of the amplitude of the second harmonic of \( v2 \) on the maximum phase error. Comparing Fig. 4(b) with Fig. 2(b) one can conclude that the modified technique has significantly reduced the maximum error in phase angle for the case when \( v2 \) is distorted by a second harmonic. Similar result was found for the case when \( v2 \)
is distorted by any other even harmonic. However, when the wave is distorted by any odd harmonic the performance of the modified technique is the same as that of the technique presented in [1]. As mentioned in [3], the error due to odd harmonic distortion cannot be canceled unless tracking selective filters are used.

Fig. 5 shows the maximum error versus amplitude of the different harmonics. To get the maximum error curves for the ith harmonic the values of $\Phi_1$ and $\Phi_i$ for wave $v_2$ was changed over the range $-180^\circ$ to $180^\circ$ at a step of $1^\circ$ at every step the error was computed, and compared with the error value recorded previously. After the comparison operation, the higher error value was recorded and the lower error value was discarded. This way the maximum error was computed for a given amplitude of the ith harmonic. The process was repeated for other values of the amplitude of the ith harmonic. In Fig. 5 the harmonic amplitude of $v_2$ is expressed as a percentage of the fundamental amplitude of $v_2$. In Fig. 5(c)–(e) it is seen that the error curves have a sharp turning point, and after the turning point the error increases rapidly. The presence of these turning points is due to the fact that the amplitude of the harmonic is too high, and it produces more than two zero crossings per cycle of the fundamental.

**V. Conclusion**

An analysis of errors associated with the digital measurement of phase angle between two signals, one of which may be distorted by a harmonic, is presented in this paper. All the results were found by running a simulation program on a VAX 11/780 computer. The results are very useful for the users of the phase meters presented in [1] and [3]. The author wants to conclude that the users of these phase meters will benefit by the results presented in this paper. Since enormous computation time is required for simulation, errors due to multiple harmonic distortion were not considered in this paper. Because of the page limitation of the publication it was not possible to show the hardware implementation of the phase measurement algorithm developed in this paper.

**References**


